QC
1
.P5812

VOLUME XII NUMBER 2

THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

LX

FEBRUARY 1901

TABLE OF CONTENTS

The Velocity of Ions Drawn from a Flame	C. D. CHILD	65
The Exact Relation between Osmotic Pressure and Vapor Pressure	ARTHUR A. NOYES	84
Dependence of the Modulus of Torsion on Tension	J. R. BENTON	100
A Determination of the Viscosity of Water	E. R. DREW	114
The Application of the Manometric Flame to the Telephone	LOUIS W. AUSTIN	121
Mounting for Tube containing Liquified Co_2	EDWIN H. HALL	125
New Books: *Thompson:* Polyphase Electric Currents and Alternate-Current Motors; *Steinmetz:* Theory and Calculation of Alternating Current Phenomena; *Oudin:* Standard Polyphase Apparatus and Systems; *Crew:* The Wave Theory of Light (Huygens, Young & Fresnel).		127

COPYRIGHT, 1901, BY THE MACMILLAN COMPANY

PUBLISHED FOR CORNELL UNIVERSITY

BY THE MACMILLAN COMPANY

LANCASTER, PA., AND NEW YORK

LONDON: MACMILLAN AND CO., LTD

BERLIN: MAYER AND MUELLER

THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS
ERNEST MERRITT, AND FREDERICK BEDELL

Two volumes of THE PHYSICAL REVIEW are published annually, these volumes beginning in July and January, respectively, and containing at least five numbers each. The price of subscription is two dollars and fifty cents a volume (five dollars a year), or fifty cents a number. Subscriptions should be sent to the publishers. THE MACMILLAN COMPANY, 41 *North Queen Street, Lancaster, Pa.*, or 66 *Fifth Avenue, New York;* Messrs. MACMILLAN & CO., LTD., *London;* or to Messrs. MAYER AND MUELLER, *Berlin.*

Previous to Volume V. (July–December, 1897) THE PHYSICAL REVIEW was published in annual volumes, each containing six bimonthly numbers, beginning with the July–August number, 1893. These may be obtained from the publishers at the former subscription price, three dollars per volume.

Subscribers who desire to have their copies of the REVIEW sent to them with the edges cut, can so obtain them by informing the publishers to that effect.

Correspondence relating to contributions should be addressed to the editors, at *Ithaca, New York.*

Manuscript intended for publication in THE PHYSICAL REVIEW must be communicated by the author; when publication in other journals is contemplated, notice to that effect should be given.

The authors of original articles published in the REVIEW will receive one hundred separate copies in covers, for which no charge will be made; additional copies, when ordered in advance may be obtained at cost.

PUBLISHED FOR CORNELL UNIVERSITY

THE MACMILLAN COMPANY

RICHARD MÜLLER-URI Braunschweig (Germany)

Manufacturer of

Blown Glass for Scientific Purposes

Reduction Apparatus

All Kinds of Lecture Apparatus and Supplies for Chemical and Physical Laboratories

Chas. R. Cross' Vacua Scale—Röntgen Tubes—McFarlan Moore Apparatus—Elster & Geitel's Transformer without Oil Insulation and Tesla Series—Spectrum Tubes (Argon, Helium) with pure Gases or Solids—Electrical Tubes after Arons, Braun, Crookes, Ebert, Geissler, Goldstein, Hittorf, Lecher, Lenard, Puluj, Zehender—Radiometers—Photo-Electric Cells and Apparatus (Selenium, etc.), Electrometer Exner.—Atmospheric Electricity Apparatus.

Sterilizer for General Work

Sole Maker and Furnisher of Patented Constructions:

115874 Röntgen-Tube for low tension currents
112835 Current Demonstrating Apparatus
58795 Perforator (Extractor)
38217 Sterilizor

Dry Pile

Apparatus for Gas-Analysis and Titration

Best Chemical Glassware

Professor Weber's Glass and Jena Normal Glass

Catalogues and Special Prospectus on Desire

The Principles of the Transformer

By FREDERICK BEDELL, Ph.D.

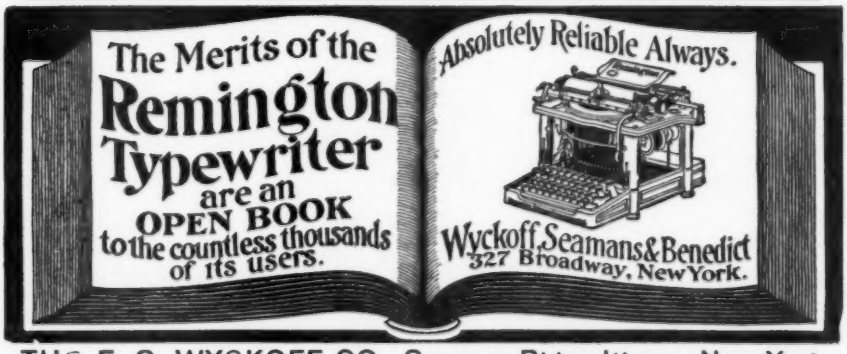
Assistant Professor of Physics in Cornell University

8vo. Cloth. Pp. ix + 416. 250 Illustrations. Price, $3.25 *net*.

THE MACMILLAN COMPANY

NEW YORK: 66 FIFTH AVENUE

BOSTON CHICAGO SAN FRANCISCO



THE E. G. WYCKOFF CO., Sprague Bldg., Ithaca, New York

WM. GAERTNER & CO.

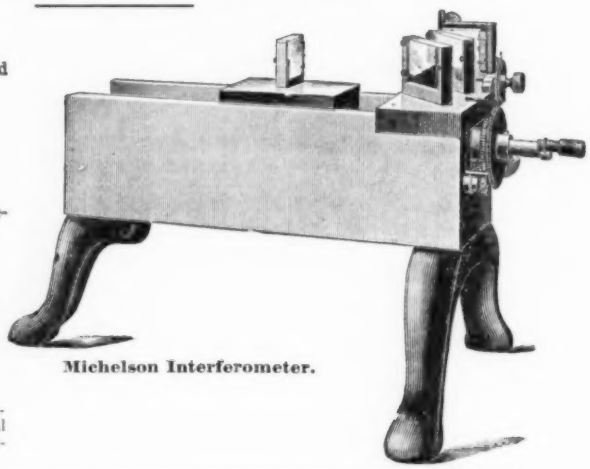
Astronomical and Physical Apparatus

5347 and 5349 Lake Avenue. CHICAGO

SPECIALTIES

Standard Apparatus of new and improved designs
Astronomical Telescopes
Spectroscopes
Micrometers
Michelson Interferometers
Reading-microscopes and Telescopes
Bolometers
Heliostats
Dividing Engines, Comparators
General Laboratory Apparatus
Universal Laboratory-supports

Special attention paid to the construction of instruments for original investigation after our own or furnished designs.



Michelson Interferometer.

PROFESSORS AND STUDENTS OF ELECTRICITY

WHILE VISITING THE PARIS EXHIBITION ARE
RESPECTFULLY REQUESTED TO PAY A VISIT TO

OLIVETTI'S STAND

(Electricity—Italian Section—Ground Floor)

They will see some

Galvanometers, Ammeters, Voltmeters

AND OTHER INSTRUMENTS

IN EACH OF THEM THERE ARE GOOD AND NEW POINTS

FOR CATALOGUES AND PRICES, ASK TO

CAMILLO OLIVETTI, Maker of Electrical Instruments, Ivrea (Italy)

WESTON

STANDARD
PORTABLE
DIRECT-READING

VOLTMETERS
MILLIVOLTMETERS
VOLTAMMETERS
AMMETERS
MILLAMMETERS
GROUND DETECTORS AND CIRCUIT TESTERS
WATTMETERS
OHMMETERS
PORTABLE GALVANOMETERS

WESTON STANDARD PORTABLE DIRECT-READING VOLTMETER FOR DIRECT CURRENT

Our Portable Instruments are recognized as The Standard the world over. The Semi-Portable Laboratory Standards are still better. Our Station Voltmeters and Ammeters are unsurpassed in point of extreme accuracy and lowest consumption of energy

---

Weston Electrical Instrument Co.

114-120 William St., NEWARK, N. J., U. S. A.

STANDARD BOOKS ON
ELECTRICITY AND MAGNETISM

The Theory of Electricity and Magnetism

By A. G. WEBSTER, A.B. (Harv.), Ph.D. (Berol.)

Assistant Professor of Physics, Director of the Physical Laboratory, Clark University, Worcester, Massachsetts

8vo. Cloth. pp. xii + 576. Price, $3.50, net

Elementary Lessons in Electricity and Magnetism

By SILVANUS P. THOMPSON, D.Sc., B.A., F.R.A.S.

New Edition With Illustrations. 16mo. Cloth. $1.40, net.

"I consider Thompson the best book of its size and nature on the subject of electricity, and have had frequent occasion to recommend it to friends and students. It is the only book that is not simply a earrangement or reproduction of the old treatise."

—WILLIAM A. HALLOCK, Ph.D., *Colmbia University, New York City*.

The Principles of the Transformer

By FREDERICK BEDELL, Ph.D.

Assistant Professor of Physics in Cornell University

8vo. Cloth. 250 Illustrations. Price, $3.25, net

This work constitutes a systematic treatise on the alternating Current Transformer, and a logical exposition of the principles involved, of the most modern methods of transformer design, construction, and testing, and of transformer systems of distribution.

A Text-Book on Electro-Magnetism

AND THE CONSTRUCTION OF DYNAMOS

By DUGALD C. JACKSON, B.S., C.E.

Professor of Electrical Engineering, University of Wisconsin

12mo. Cloth. Price, $2.25, net

Alternating Currents and Alternating Current Machinery

By DUGALD C. JACKSON, C.E. and JOHN P. JACKSON, M.E.

University of Wisconsin *Pennsylvania State College*

12mo. Cloth. pp. xvii + 729. Price, $3.50, net

"I like the way in which they develop their theory and make practical applications. I feel sure that the book will receive a hearty welcome."

—GEO. W. PATTERSON, Jr., *Asst. Professor of Physics, University of Michigan.*

The Storage Battery

A PRACTICAL TREATISE ON THE CONSTRUCTION, THEORY AND USE OF SECONDARY BATTERIES

By AUGUSTUS TREADWELL, Jr.

Cloth. 12mo. pp. xx + 257. Price, $1.75, net

Electricity in Town and Country Houses

By PERCY E. SCRUTTON.

Second Edition. Fully Illustrated. 12mo. Cloth. pp. xii + 148. Price, $1.00

A brief and simple statement of the first principles of the application of electricity to the needs of the home, for heating, cooking, lightning, etc. A popular and untechnical book, freely illustrated.

THE MACMILLAN COMPANY

New York: 66 Fifth Avenue

BOSTON CHICAGO SAN FRANCISCO

The

General Electric Company's

Pocket Ammeters

INCLINED COIL POCKET AMMETER (Covered)

of twenty-five amperes' capacity can be used upon either direct or alternating current, and gives accurate readings for experimental work in the laboratory.

INCLINED COIL POCKET AMMETER (Uncovered)

This pocket ammeter is one of a complete set of portable inclined coil electrical measuring instruments, for use in the laboratory, central station and outside work.

GENERAL ELECTRIC COMPANY

Main Office: SCHENECTADY, N. Y.

Sales Offices in All Large Cities

For CANADA: Address the CANADIAN GENERAL ELECTRIC COMPANY, Toronto, Can.

QUEEN INSTRUMENTS

New Queen Wall Pattern Galvanometer

The **Moving Coil** in this **Galvanometer** is of **small diameter** and light weight. It can be made **dead beat** and will then give **readings** with much greater **rapidity** than other forms on the market. When arranged for **Ballistic** work the **period** of **swing** can be made as desired.

A circular issued September 1st describes in detail this and a number of other new pieces of apparatus. If interested you are requested to ask for circular 500.

PHYSICAL APPARATUS INDUCTION COILS
ELECTRICAL TESTING INSTRUMENTS

QUEEN & CO., INC.

J. G. GRAY, Pres.

Electrical and Scientific Instruments

50 Fifth Avenue, New York
480 Monon Bldg., Chicago

1010 CHESTNUT ST., PHILADELPHIA

Volume XII. *February, 1901.* *Number 2.*

THE

PHYSICAL REVIEW.

THE VELOCITY OF IONS DRAWN FROM A FLAME.

By C. D. Child.

A NEW method for determining the velocity of ions in a gas was suggested by Prof. J. J. Thomson in a recent number of the *Philosophical Magazine*.[1] As far as the present writer is aware this has not been used experimentally. It appeared more satisfactory than any method which had previously been used, and an actual test of it seemed desirable. Moreover, it appeared as if a simplification of Professor Thomson's suggestion could be used to advantage in studying the velocity of ions drawn from an electric arc, and it was deemed wise to apply this simplification in the first place to a case that could be made to conform as closely as possible to the theoretical conditions and that could be better tested by comparison with other data. The test was accordingly made by applying the method to the determination of the velocity of ions produced by a flame.

No attempt has been made to secure great accuracy, since the quantity to be measure, *i. e.*, the velocity of the ions, is one depending on a great variety of conditions, and since this work is rather the test of a method than anything else. However, the accuracy of the results here given will compare most favorably with that hitherto secured in attempting to measure such velocities.

The Method Employed.—The method consisted in measuring the potential and the flow of current in a space through which only ions charged with one kind of electricity were passing, and in com-

[1] Phil. Mag. (5), 47, p. 265.

puting from these measurements the velocity of the ions. It is described by Professor Thomson as follows:

"A very convenient method of determining the velocity of the ions and one which can be applied in nearly every case of conduction through gases, is to produce the ions in one region and measure the electric intensity at two points where there is no production of ions, but to which ions of one sign only can penetrate under the action of the electric field. Thus let A, B, Fig. 1, represent two parallel plates immersed in a gas, and let us suppose that in the layer between A and the plane LM we produce a supply of ions, by any means, and suppose that the gas between LM and B be screened off from the action of the ionizer. Then if A and B are connected to the poles of the battery, a current will pass through the gas, and this current in the region between LM and B will be carried by ions of one sign only. These will be positive if A is the positive pole, negative if it is the negative. Let us find the distribution of electric intensity in the region between LM and B. Let us suppose A is the positive plate. Then all the ions in this region are positive.

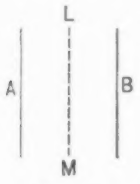


Fig. 1.

Let X be the value of the electric intensity at a point, e the charge carried by each ion, n_1 the number of positive ions, k_1 the velocity of positive ions for unit potential gradient, and x the distance from LM, and we have $\frac{dX}{dx} = 4\pi n_1 e$ and $k_1 n_1 X e = i$ where i is the current through unit area; from these equations we have,

$$X\frac{dX}{dx} = \frac{4\pi i}{k_1} \text{ or } X^2 = \frac{8\pi i}{k_1}x + C^2. \qquad (1)$$

Hence if we measure the value of X at two points in the region between LM and B, and also the value of i, we can from this equation, deduce the value of k_1 and hence the velocity of the positive ions in a known electric field. To determine the velocity of the negative ion we have only to perform a similar experiment with the plate A negative."

Objection to Determining the Potential with a Wire.—The more difficult part of the work is to determine accurately the potential.

Two methods have been used for this purpose. A wire has been placed at the point to be investigated, as was done by Zeleny,[1] or a water dropper may be used as in some experiments performed by the present writer.[2] There is the general objection to the use of the wire that it takes a very long time for it to assume the potential of the surrounding space but this is not usually a very serious objection. There is, however, in this particular case a much more serious one.

A body surrounded by a gas can not discharge through the gas, unless there are ions in the gas which are charged with electricity of the opposite sign from that on the body itself. As far as is known ions do not pass from a solid body into gas. They only pass from the gas and give up their charge to the solid. We may perhaps express this idea by saying that the charge can not be driven off from the body. It can only be neutralized by the opposite kind of electricity being attracted to it. Thus in the space *LMB*, Fig. 1, there are only positive ions, if we imagine *A* to be the positive pole. Consequently if a wire which has a negative potential as compared with the surrounding region is placed in this space, it will attract the positive ions and acquire a higher potential. If, on the other hand, it has a positive potential as compared with the surrounding region, it will repel the positive ions and since there are no negative ions to be attracted, it cannot change its potential. As a result of this if there is the slightest unintended leak of positive electricity on to the wire, this electricity will not be able to escape and the wire will register a higher potential than it should.

This unintentional leak of positive electricity on to the wire was especially liable to happen in this case, since the flame producing the ions in the space *LMA* is charged positively, and everything in the room soon became thus charged. It would seem an easy matter to protect the wire from this positive electricity, but in practice it was found difficult to get rid of the last trace of it, and this last trace was entirely sufficient to change the readings. Readings taken with the wire were found to vary from day to day to such an extent that the method was finally abandoned.

[1] Phil. Mag. (5), 46, p. 47.
[2] PHYS. REV., 6, p. 285.

The idea that the positive charge will not leak off from the wires while a negative one will, is not simply what might be expected from theory. Observation shows the same. If a wire placed in the space *LMB* is charged a few volts lower than the potential of the surrounding space as shown by other methods, it quickly assumed a higher potential. If, on the other hand, it was charged a few volts higher, the change in potential was exceedingly slow and appeared to depend entirely on leakage at other points along the wire rather than on any in the region *LMB*. So clearly was this shown that it afforded a proof of the correctness of the general conception as to how discharge is produced under these condition.

Use of the Water Dropper for Determining Potential.—For this reason the water dropper was used to find the potential in *LMB*. Even with this it was by no means easy to avoid errors due to the charged air of the room. The wires were all placed inside of metal pipes. The switches were placed within a tin box, and were operated by cords passing through small openings. The water dropper itself was entirely surrounded by tin, with the exception of the tube through which the water passed. This was 10 cm. long and 2 mm. in diameter and was placed parallel to the plate B, so that the potential along its whole length was as nearly as possible the same as that at the opening at the bottom. The space between this tube and the metal protecting the water dropper was closed by sealing wax.

The ions from the flame penetrate far more than was to be expected. For a long time it was thought that the water dropper did not register the same potential for different rates of flow, and indeed the writer undertook a series of measurements to show what the relation was between the rate of flow and the potential registered. It was found, however, that enough charged air when carried by the draught from the flame would pass through very small openings to make a difference in the readings of the water dropper, especially when the water was dropping slowly. When at last the water dropper was so insulated that there was no leak onto it, it registered the same potential whatever the rate of flow. This may be offered as proof of the correctness of the determination of potential.

Arrangement of Apparatus.—Fig. 2 gives an outline of the arrangement. A and B are two plates between which the discharge occurs. D is the part of the water dropper between the plates. F is the flame producing the discharge. A and B are connected through the reversing key K to the water battery H. Any point of the battery could be connected to the ground at G. D was connected to the electrometer E.

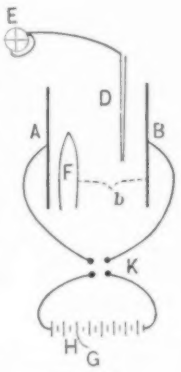


Fig. 2.

The distance from the flame to B will be designated by b. This corresponds to the distance in Fig. 1 between LM and B. The difference of potential between the flame and B will be designated by V_b.

The potential of the point D was found by varying H on the battery until the water dropper came to have zero potential. The difference of potential between F and H was then noted. This kept D and its connections at the same potential as the ground, and avoided any error due to leakage from the water dropper.

The water dropper was placed on a dividing engine so that its position could be easily and accurately changed.

Since the flame F was close to the plate A, it assumed nearly the potential of A and it was found that the potential of F was kept more constant by placing in the flame a wire connected directly to A. Different parts of the flame have, of course, slightly different potentials, but this cause of error could not well be avoided, and in any case the error thus introduced was smaller than other unavoidable errors.

The plates A and B were tin plates 20 cm. in diameter, fastened to boards so as to present to the flame as plane surfaces as possible. The flame was furnished by a row of Bunsen burners 2 cm. apart. These could not be said to ionize a space bounded on one side by an *infinite* plane as LM in Fig. 1, but this arrangement seemed as near an approach to that condition as could well be devised, and apparently satisfied the requirements.

A water battery was used because it was the only means of producing the required potential which the writer had at his command.

It was kept carefully insulated and was found to give practically the same potential on different days. It was calibrated whenever it was used by comparing parts of it at a time with a storage battery and the voltage of the storage battery was measured by a Weston voltmeter.

The electrometer was one designed by Elster and Geitel.[1] For measurents of small potentials it was connected heterostatically and the needle was charged by a water battery. For higher potentials it was connected idiostatically. It was calibrated by comparison with a Weston voltmeter.

Sources of Error.—The observations were rendered inaccurate to a small extent by the movement of the flame and also by the changes in the flame produced by variations in the quality of the gas. Care was taken to keep the flame as constant as possible, and it is believed that these sources of error had but very little effect on the final result.

It was also impossible to determine accurately the boundary between the flame and the surrounding atmosphere, or more correctly between the region where there were ions of both signs and that where there were ions of one sign only. It is not necessary to know where this boundary is in order to use the method as given by Professor Thomson but it is necessary in order to use a modification of it to be described later.

It was found that the gas for about 1 cm. to the right of the visible part of the flame had very nearly the same potential as that of the flame itself, any difference of potential being certainly less than one volt. It is not surprising that this should be so, since the hot air from the lower portions of the flame spreads itself for some distance about the flame proper. This can be easily shown by passing the light from an electric arc through the flame of a Bunsen burner onto a screen behind the burner. The stream of hot air will show itself by the change of refractive index of the air. This region of hot air about the flame was found to coincide practically with that in which the potential was the same as that of the flame.

The method used for finding the boundary between the ionized and unionized regions was to find with the water dropper the point

[1] Weid. Ann, 64, p. 680.

where the potential began to vary from that of the flame. This is not a very satisfactory method, but the only other one possible was to observe the outline of the flame as mentioned above. This later method did not give any more definite results due to the flickering of the flame, and when it was used there remained always the uncertainty as to whether the boundary sought was exactly the same as that shown in the image of the flame on the screen.

Comparison of Potentials with Flame Positively and Negatively Charged.—Table I. gives the result of a set of readings, first when F was charged positively, and second when charged negatively. These are not the average of several sets of readings, but the readings as they were taken down in a single set; b was 8 cm., V_b was 98.5 volts. Column 1 gives the distance from the flame, column 2 the potential in volts below the flame when the flame was charged positively, and column 3 that above the flame when it was negative.

TABLE I.

1 Distance from Flame.	2 Potential with Flame. +	3 Potential with Flame. −	1 Distance from Flame.	2 Potential with Flame. +	3 Potential with Flame. −
1	2	3	5	49	49
2	13	12	6	64	64
3	23	22	7	81	79
4	36	36			

The differences in the two sets of readings were no greater than are apt to occur in taking two sets with F charged alike in both cases. In fact the results of many trials showed that there was no difference which could be detected with certainty between the form of the curves of potential given when F was positive and that when F was negative. Consequently only the potential taken with F positive are given in the following tables:

Potentials with Different Distances between the Flame and B.—Table II. gives a series of readings taken with different values of b, V_b being in all cases 98.5 volts.

Column 1 gives the distance from the flame. Columns 2, 4 and 6 are observed values of the potential in volts below that of the flame, and columns 3, 5 and 7 are corresponding values computed

TABLE II.

1 Distance from Flame.	2 Observed Potentials.	3 Computed Potentials.	4 Observed Potentials.	5 Computed Potentials.	6 Observed Potentials.	7 Computed Potentials.
1	3	4.4	7	6.7	12	12.3
2	13	12.3	20	19.0	34	34.8
3	23	22.6	36	34.8	64	63.9
4	36	34.8	55	53.6		
5	49	48.7	76	74.9		
6	65	63.9				
7	81	80.6				

by a method to be described later. Columns 2 and 3 are values when b was 8 cm., 4 and 5 when it is 6 cm., and 6 and 7 when it was 4 cm.

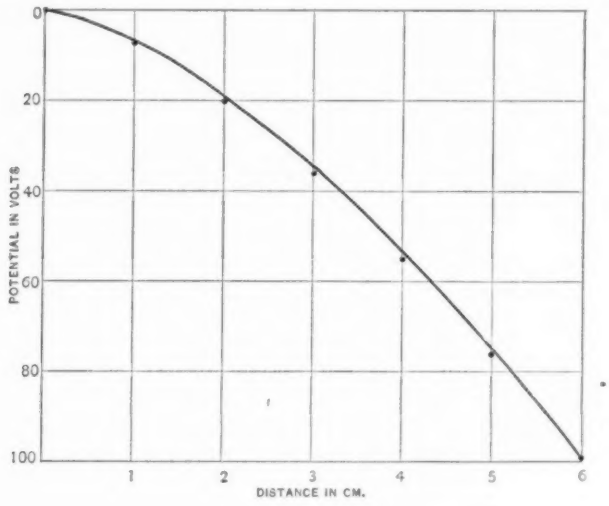


FIG. 3.

The data given in columns 4 and 5 are represented in Fig. 3. The smooth curve represents the theoretical values of the potential, and the points by the side of this line the observed values.

Potential with Different Differences of Potential.—Table III. gives the readings with b always the same, but with different values for V_b, b was 6 cm. Column 1 gives the distance from the flame; columns 2 and 3 the observed and calculated potentials below that of the flame when V_b was 50.5 volts; columns 4 and 5 the potentials for $V_b = 98.5$ volts; columns 6 and 7 for 198.5 volts.

TABLE III.

1 Distance from Flame.	2 Observed Potentials.	3 Computed Potentials.	4 Observed Potentials.	5 Computed Potentials.	6 Observed Potentials.	7 Computed Potentials.
1	4	3.4	7	6.7	16	13.5
2	10	9.7	20	19.0	37	38.2
3	19	17.9	36	34.8	70	70.2
4	28	27.5	55	53.6	108	108.5
5	39	38.4	76	74.9	152	151.0

Determination of Current.—The current between the flame and *b* was found by replacing the plate which had previously been at *B* by one which was 5.6 cm. in diameter, surrounded by a guard ring. The inner diameter of the guard ring was 6.4 cm. and the outer 20 cm. *B* was placed on a dividing engine so that its position could be easily changed.

When the current was large it was measured by a galvanometer, when small, by an electrometer by noting the rate at which a condenser became charged. The capacity of the condenser was found by the use of a ballistic galvanometer. The constant of the galvanometer thus used was computed from its constant when used as a tangent galvanometer. As a check upon this method the capacity was also found from observations of the constant deflection produced in the galvanometer by discharging the condenser through it a given number of times a second. The constant of the galvanometer when used as a tangent galvanometer was found by comparing it with a Weston voltmeter.

The constant of the galvanometer thus used was 4.45×10^{-8} ampère per scale division. The period of the needle was 12.6 sec. making the constant for ballistic work 8.93×10^{-8} coulomb per scale division. The capacity of the condenser was 5.05×10^{-3} microfarad.

The rate of leak of the condenser was small as is shown by the following: The condenser with all of its connections as they were to be used in the determination of the current were charged and the change in potential due to unavoidable leakage noted. This change is shown in Table IV. The leakage is so small that it may be neglected.

TABLE IV.

Time in min.	0	1	2	3	4	5	6
Potential in volts	8.34	7.88	7.62	7.4	7.24	7.06	6.96

Two different sizes of plates were used at B and it was found that the current flowing on to them was proportional to the size of the plates. Whatever variations there were from this rule appeared to be due entirely to errors of observation, so that only one size of plate was used in the measurements here given.

The current was found from the electrometer as follows: B was connected to the electrometer in multiple with the condenser. First it was grounded. The connection to the ground was broken, and after a given length of time the connection to B was broken. The potential of the electrometer was noted and this divided by the length of time equals the rate at which B becomes charged. This was repeated several times and with different lengths of time. The readings taken from time to time gave results which varied but a few per cent. The potentials were found from the readings by the calibration curve of the electrometer. The results from the averages of these readings are given. The change in potential of B was so small when compared with the total difference of potential between B and the flame that it was neglected.

In two cases the values of the currents found by the electrometer were compared with readings found by the galvanometer. When b was 2 cm. and V_b was 98.5 volts and the flame charged positively the deflection of the galvanometer was .15 which was equivalent to 6.55×10^{-9} ampère. The change in potential of the electrometer in 4 sec. was 5.2 volts. The current is equal to the capacity times the voltage divided by the length of time. This gives 6.5×10^{-9} ampère. Similarly the value of the current when the flame was charged negatively as found by the galvanometer was 7.85×10^{-9} ampère and as found by the electrometer 7.4×10^{-9} ampère. It was not possible to be certain concerning the reading of the galvanometer in the last figure. So that the fact that these values so nearly coincide is partly accidental.

Current with Different Distances.—Table V. gives the amount of current flowing with different values for b; V_b in each case being

98.5 volts. Column 1 gives the different distances. Column 2 the current when the flame was positive and 4 when it was negative. It may be noticed that the current varies approximately inversely as the cube of the distance. Columns 3 and 5 give the values calculated on this assumption assuming also that the values given in the table for 2 cm. are correct. The theoretical reason for this law will be discussed later. These values are not for i as given in the formulæ, but for the total current flowing on to the plate 5.6 cm. in diameter.

TABLE V.

1 b	2 Observed Current with Flame. +	3 Computed Current with Flame. +	4 Observed Current with Flame. —	5 Computed Current with Flame. —
1	$652. \times 10^{-10}$	$521. \times 10^{-10}$	$1065. \times 10^{-10}$	$610. \times 10^{-10}$
2	65.2 "	65.2 "	76.2 "	76.2 "
3	7.9 "	8.15 "	9.3 "	9.5 "
4	2.12 "	2.41 "	2.43 "	2.82 "
5	.65 "	1.02 "	.91 "	1.19 "

The current for 1 cm. was measured with the galvanometer, for 2 cm. with both the galvanometer and the electrometer, the average values being given. The currents for the other distances were determined with the electrometer.

Current with Different Differences of Potential.—Table VI. gives the amount of current for different values of V_b, b in each case being 6 cm. Column 1 gives V_b, column 2 the current when the

TABLE VI.

1 V_b	2 Observed Current with Flame. +	3 Computed Current with Flame. +	4 Observed Current with Flame. —	5 Computed Current with Flame. —
50.5	$.50 \times 10^{-10}$	$.51 \times 10^{-10}$	$.52 \times 10^{-10}$	$.58 \times 10^{-10}$
98.5	2.12 "	2.12 "	2.43 "	2.43 "
198.5	8.8 "	8.6 "	9.05 "	9.25 "

flame is positive, column 3 when it is negative. In this case the current varies nearly as V_b^2. Columns 3 and 5 are computed on this assumption.

This law seemed important enough to test again, especially since the deflection in the electrometer was here so small that an error in the last figure of the reading would produce a large percentage of error. A similar set of readings was accordingly taken with b equal to 4 cm. These readings and the computed values are given in Table VII.

TABLE VII.

1 V_b	2 Observed Current with Flame. +	3 Computed Current with Flame. +	4 Observed Current with Flame. −	5 Computed Current with Flame. −
50.5	1.87×10^{-10}	1.9×10^{-10}	2.5×10^{-10}	2.48×10^{-10}
98.5	7.9 "	7.9 "	10.3 "	10.3 "
198.5	32. "	30. "	40.3 "	39. "

All of these readings in the last two tables were taken with the electrometer.

Calculation of k.—If in equation (1) we let x_1 and x_2 be two particular values of x, and X_1 and X_2 corresponding values of X, we have

$$X_2^2 - X_1^2 = \frac{8\pi i}{k_1}(x_2 - x_1)$$

or

$$k_1 = \frac{8\pi i(x_2 - x_1)}{X_2^2 - X_1^2}.$$

This equation is expressed in the electrostatic system. The values given in the tables are given in the practical system. The above equation when expressed in this system becomes

$$300k_1 = \frac{8\pi i \times 3\times 10^9(x_2 - x_1)}{(\frac{1}{300})^2(X_2^2 - X_1^2)}$$

or

$$k_1 = \frac{72\pi i \times 10^{11}(x_2 - x_1)}{X_2^2 - X_1^2}. \qquad (2)$$

If we take the case when the total distance b is 6 cm. and V_b is 98.5 volts, if x_1 and x_2 are equal to 1 and 5 respectively we find from Fig. 3 that $X_1 = 9.8$ and $X_2 = 22$. To find the current per unit cross-section the currents given in Table V. must be divided by

the area of the plate B. This was $2.8^2\pi$. This gives the value for i in this case to be $.89 \times 10^{-11}$. Substituting these values in (2) we have $k_1 = 2.08$ cm. per second for a volt per cm.

Wilson[1] found the velocity of the positive ions in the flame itself to be in some cases as high as 50 cm. per second, and that of the negative 1000 cm. per second. McClellan[2] found the velocity of the positive ions in the hot air above the flame to vary from .22 cm. to .03 cm. per second.

It is evident that in finding X_1 and X_2 graphically we are liable to introduce an error due to a wrong estimate of the tangent to the curve. This is more serious since the denominator of the fraction in (2) is the difference between the squares of two quantities each liable to such an error. Moreover, it seemed to be impossible to find the potential at any point nearer than one volt, and an error of one volt in the potential affected the tangent to the curve quite materially. It was estimated that it would be quite possible for an error of ten per cent. due to this cause to occur.

Simplification of the Method.—There is, however, another way in which k_1 can be computed which will now be considered. It is seen that the potential gradient in Fig. 3 at the flame is quite small. The explanation of this appears to be as follows: The number of ions in the flame is exceedingly large compared with the number actually drawn toward B. When this is the case the amount of discharge is not limited by the number of ions in the flame, but by the change in potential in LMB produced by the ions there.

We may show this mathematically as follows: If V equals the potential at any point in LMB and $a = \frac{8\pi i}{k_1}$, we may write equation (1)

$$\frac{dV}{dx} = -\sqrt{ax + C^2}$$

where C is the value of $\frac{dV}{dx}$ at the origin. The negative sign before the radical corresponds to the case imagined, the plate B being negative, the flame positive. Integrating this equation we have

[1] Proc. Roy. Soc., '1899.
[2] Phil. Mag., 46, 41.

$$V = -\frac{2}{3a}\{(ax + C^2)^{3/2} - C^3\} \qquad (3)$$

if $V = 0$ at the origin.

If we place the origin at the edge of the flame, *i. e.*, at LM in Fig. 1, C is nearly equal to zero as is shown graphically in Fig. 3. This may also be shown from theoretical considerations. If N_1 is the number of positive ions at LM equation $k_1 n_1 X\varepsilon = i$ becomes $k_1 N_1 C\varepsilon = i$. If we substitute this value for i in equation (3) we get an equation of the fourth degree in C. We can not solve this algebraically, but we can get an approximate value if we assume C to be small, and the experimental work has shown that this is a safe assumption to make. Substituting this value for i in equation (3), making this assumption, remembering the $a = \frac{8\pi i}{k_1}$ and that if C is small, N_1 must be large, we find as an approximation that

$$C = \frac{9V^2}{32\pi x^3 N_1 \varepsilon}.$$

We can get some idea of the value of $N_1\varepsilon$ from the work which has been done on the conductivity of flames. Thus in the flame $i = N_1\varepsilon X(k_1 + k_2) = \frac{X}{\rho}$, if we assume the number of positive and negative ions to be the same, and if ρ is the resistance of a cube 1 cm. on each side. Or $N_1\varepsilon = \frac{1}{\rho(k_1 + k_2)}$. Data is given by Marx[1] from which a value for ρ for an ordinary Bunsen burner flame may be computed. From his data we find that we will not be far wrong to assume $\rho = 3 \times 10^6$ ohms or $\frac{1}{3} \times 10^{-5}$ E. S. unit. It will also be safe to say that $(k_1 + k_2)$ in E. S. units is not greater than 3×10^5 which gives 1 for the value of $N_1\varepsilon$.

If $V_b = 100$ volts when $b = 6$ cm., we have $C = 4.6 \times 10^{-5}$. The value for a in the same case and in the same units is 1.16×10^{-3}. The ratio of C^2 to ax when $x = 1$ is 1.83×10^{-6}, that is, any error which would be caused by dropping C would be less than .0002 per cent. when $x = 1$ cm., and would be still smaller as x becomes greater.

[1] Drude Ann., 2, p. 787.

All of this is, of course, only a rough approximation, but it will serve to show that in the case of discharge from a flame C is so small that we may safely neglect it in equation (3).

If we place $C = 0$, equation (3) becomes

$$V = -\tfrac{2}{3}\, a^{\frac{1}{2}}\, x^{\frac{3}{2}} \qquad (4)$$

If we plot this giving to a such a value that $V = 98.5$ volts when $x = 6$ cm., we will have the smooth curve drawn in Fig. 3. The points which were found from observation were all near to the curve. Values of V computed in this same way are given in columns 3, 5 and 7, Table II. and columns 3, 5 and 7, Table III.

In all these cases there are no great differences between the observed and the calculated values. We therefore consider ourselves justified in supposing that in these cases the value of $\frac{dV}{dx}$ at the origin is practically zero. Substituting the value of a in equation (5) and solving for i we have $i = \frac{9V^2k_1}{32\pi x^3} = \frac{9V_b^2k_1}{32\pi b^3}$ when $x = b$. This gives a theoretical basis for the fact already noticed that the current varies approximately as $\frac{V_b^2}{b_3}$. The deviations from this rule will be discussed shortly.

If we have an unlimited supply of ions in the flame so that equation (4) holds good, and if we can measure V_b together with i, we can compute k_1 without measuring the potential in the space LMB. We thus do away with the error arising from mistaking the tangents to curve on Fig. 3. However, we introduce another source of error in most cases, due to the difficulty of determining the boundary LM. But in the case of the flame when the distance b is 4 cm. or more, this error is at least not greater than the other, and the latter method of computing k_1 is very much simpler.

If we express the quantities in the practical system the above equation becomes

$$k_1 = \frac{32\pi i \times 10^{11} b^3}{V_b^2}.$$

Table VIII. gives a series of values for k_1 and k_2, the velocity of positive ions for unit potential gradient, which have been computed by this formula from Table V. where V_b is 98.5 volts.

TABLE VIII.

1 b	2 Current with Flame. +	3 Current with Flame. −	4 k_1	5 k_2	6 k_2/k_1
1	275×10^{-11}	450×10^{-11}	2.85	4.66	1.64
2	27.5 "	32.1 "	2.27	2.65	1.17
4	13.34 "	3.92 "	2.21	2.60	1.18
6	.893 "	1.08 "	2.00	2.41	1.20
8	.274 "	.384 "	1.45	2.03	1.40

Column 1 gives the distance b; column 2 the current for unit area when the flame was positive; column 3 when it was negative. Columns 4 and 5 give corresponding values for k_1 and k_2. Column 6 gives the ratio between these values.

Variation of k.—We see here a very decided tendency for the values of k_1 and k_2 to become smaller as the distance becomes greater. Two explanations for this may be suggested. One is that the ions move more slowly the longer the time that elapses after they leave the flame. The other is that the apparent change in velocity is due to errors of measurement, that the large values at first are due to an error in the estimate of the position of the boundary about the flame, and that the smallness of the current when the distance was 8 cm. was due to the fact that with increase of distance between the flame and B the finite size of the plates and flame comes to produce a very large error.

Let us consider the time that actually elapses during the passage of the positive ions from the flame to B, assuming that k_1 is constant and that $V = -\frac{2}{3}a^{\frac{1}{2}}x^{\frac{3}{2}}$. Let T_1 equal this length of time.

$$T_1 = \int_0^{T_1} dt = \int_0^b \frac{dx}{k_1 \frac{dV}{dx}} = -\int_0^b \frac{dx}{k_1 a^{\frac{1}{2}} x^{\frac{1}{2}}} = -\frac{1}{2}\frac{b^{\frac{1}{2}}}{k_1 a^{\frac{1}{2}}} = \frac{b^2}{3k_1 V_b}.$$

Similarly T_2, the time that it takes for the negative ions to pass from the flame to B, equals $\frac{b^2}{3k_2 V_b}$. This will also be correct when we express the quantities in the practical system. For example, if $b = 4$ cm., $V_b = 98.5$ and $k_1 = 2.26$ cm. per sec. per volt, $T = .024$ sec.

We see that the time becomes great when b is increased and also when V_b is diminished, and changing the values of V_b does not introduce any of the possible errors which have been suggested as causing the changes in the values of k_1 and k_2.

Accordingly the values of k_1 and k_2 when V_b varies are given in Table IX. These values are computed from the data of Tables VI and VII.

TABLE IX.

1 V_b	2 Current with Flame. +	3 Current with Flame. −	4 k_1	5 k_2
50.5	$.21 \times 10^{-11}$	$.232 \times 10^{-11}$	1.78	1.86
98.5	.893 "	1.08 "	2.00	2.41
198.5	3.7 "	4.04 "	2.04	2.23
50.5	$.787 \times 10^{-11}$	$.945 \times 10^{-11}$	1.92	2.39
98.5	3.34 "	3.92 "	2.21	2.60
198.5	13.5 "	16.5 "	2.2	2.76

Column 1 gives the values of V_b. The other columns given are the same as in Table VIII. The first three values for k_1 and k_2 are derived from Table VI where b was 6 cm.; the last three from Table VII where b was 4 cm.

We see that there appears to be a slight decrease in k_1 and k_2 as the differences of potential become smaller. The time varies inversely as the *first* power of V_b but directly as the *square* of b, so that we have a variation in T_1 by changing b from 1 cm. to 8 cm. which is sixteen times greater than that secured by changing V_b from 50.5 to 198.5 volts. Table IX is at least in harmony with the idea that k_1 and k_2 decrease as T_1 and T_2 increase.

If there were any great variation in k_1 it would show itself in the curve in Fig. 3. Thus if k_1 instead of being a constant decreases as T_1 increases, the values for the potential would be somewhat higher than they are. For instance if $k_1 = k_1' \left(1 - \frac{t}{15}\right)$ where k_1' is a constant the values for V which we would have in Fig. 3 when $x = 4$ cm. would be 55.9 instead of 53.6. In fact the values of the potentials in Tables III., IV., and V. are slightly larger than the theoretical values. While this by itself would mean little we can say that this also is in harmony with the other data.

What is more decisive than anything else, is that we know from the work of others that the velocity of ions in the flame is very much greater than that of the ions in the hot air above the flame, and consequently a change must take place at just the region which is being investigated. The writer is, therefore, of the opinion that the change in velocity as b is moved from 1 cm. to 6 cm. is largely a real change, but that the change in going from 6 cm. to 8 cm. is due rather to lack of conformity between the actual and ideal conditions, since there does not seem to be any reason why so large a change should occur at that place.

If this conclusion is correct none of the equations which have been given are strictly true, for it was tacitly assumed that k_1 was a constant. No doubt k_1 always varies more or less and the equations are to a certain extent incorrect, but they at least give a close approximation, and it is not probable that k_1 would often vary as rapidly as it does in this case.

Simplification Applied Qualitatively.—The simplification here suggested will also enable one to tell something about the *relative* velocities of the positive and negative ions in cases which can not be considered analytically. In order to determine this relative quantity in the case just considered it was only necessary to compare the rate of discharge when B was positive with that when B was negative. In fact that is all that is necessary in any case of discharge from a place where there is an unlimited supply of ions of both kinds, whether the surfaces are of such a shape that they can be treated analytically or not.

Let us consider the case where LM and B in Fig. 1 are surfaces of any shape. If we imagine the current to be in the beginning zero, as it increases the number of positive ions in LMB increases, the potential in this region increases, and the fall of potential at LM decreases until finally the fall of potential at LM is so small that only a sufficient number of ions are drawn past LM to maintain the current.

The current would not be limited by the number of ions in the flame, but it would be choked, due to the number of ions present in the LMB. And this number would be greater for a given current the slower the movement of the ions. If under similar conditions

the current is greater with positive ions in LMB than with negative ones, it shows that the velocity of the positive is the greater, and vice versa.

This holds where LM is a plane, a cylinder as with a single Bunsen burner, or a sphere as with an arc. It holds whether LM is a regular surface or not. The modification of the mathematical work to the cases of the cylinder and the sphere is entirely feasible in case it may be desired.

Comparison of this Method with Other Methods.—The disadvantages of this method have been apparent in the preceding description. If one uses the method as first given by Professor Thomson, there must ever be more or less uncertainty in determining the tangent to the curve of potentials, and this error would become more serious as the supply of ions in ALM becomes smaller. For then the fall of potential in LMB would become more nearly uniform, X_1 and X_2 would be nearly the same, and any error in either would produce a much larger error in the difference of their squares.

The modification suggested here is generally liable to error due to uncertainty as to where LM is, and moreover it cannot be used at all when the source of ions in LMB is small. The greatest source of error in the preceding work, *i. e.*, changes in the position and character of the flame is not one which is in any way due to the method used.

Summary.—We may summarize the preceding paper as follows:

First, an application of the method suggested by Professor Thomson shows that the velocity of the positive ions drawn from a Bunsen burner is approximately 2.2 cm. per sec. for potential gradient of one volt per cm., and that for negative ions 2.6 cm.

Second, in the case of an unlimited supply of ions, if the discharge takes place between two regular surfaces the velocity may be determined by simply measuring the current per unit area and the difference of potential between these surfaces, and if the surfaces are not at all regular, the relative velocities of the positive and negative ions may be determined by comparing the positive and negative currents.

A description will be given in another article of an application of this simplified method to the comparison of the velocities of the positive and negative ions drawn out from an electric arc.

COLGATE UNIVERSITY, Nov. 21, 1900.

THE EXACT RELATION BETWEEN OSMOTIC PRESSURE AND VAPOR PRESSURE.

BY ARTHUR A. NOYES.

1. INTRODUCTION.

THE earlier thermodynamical derivations of the relation between osmotic pressure and vapor-pressure were inexact, inasmuch as the influence of the compression of the liquid was neglected. From the failure to take this influence into account, has resulted in my opinion an erroneous conception of the volume-factor by which the osmotic work must be divided in order to give the osmotic pressure. Two years ago, Abbot and myself[1] published a derivation based on the consideration of an osmotic column which appeared to be free from these imperfections. In a later publication[2] I still further discussed the above-mentioned volume-factor. Dieterici[3] in a recent article, has, however, raised certain objections to my views, and to the derivation of the relation by means of the osmotic column. His remarks, so far as they are of a personal nature, need not be considered. It would also be unprofitable for me to enter into an immediate discussion of the scientific considerations in Dieterici's article. In order to make the matter entirely clear, it seems, instead, necessary to deduce anew the relation in question, taking into account the relatively small factors which are involved. Incidentally, I will discuss those remarks and objections of Dieterici which seem to be of importance.

2. DEFINITION OF OSMOTIC PRESSURE.

Before deriving the relation between vapor-pressure and osmotic pressure, it is necessary to have a clear and precise conception of

[1] This REVIEW, 5, 113.

[2] Die Beziehung zwischen osmotischer Arbeit und osmotischem Druck, Ztschr. für physik. Chem., 28, 220.

[3] Ztschr. für physik. Chem., 29, 139.

the latter property. We define osmotic pressure to be the pressure exerted on a semipermeable wall which separates a solution from its solvent. In order that the two may be in equilibrium with each other and no passage of solvent through the semipermeable wall take place, it is necessary that the solution be subjected to a greater external pressure than the solvent; the difference in the two external pressures is then equal to the osmotic pressure, and a measure of it. It is to be noted that this pressure-difference, or the osmotic pressure of the solution, will, like vapor-pressure, vary with the absolute value of the external pressure or with the degree of compression of the solution, as well as with the temperature.

To make this definition clearer, consider two special cases which will be of importance in the subsequent considerations. In the figure, let A be a solution and B a solvent separated from each other by a fixed semipermeable wall EE, and enclosed by pistons C and D on which any desired pressure can be exerted. First, suppose that the pressureon piston D is made equal to the vapor-pressure p_0 of the solvent. Then to produce equilibrium, a pressure of $P_1' + p_0$ must be exerted on piston C, where P_1' is the osmotic pressure exerted on the wall E by the *compressed* solution. It is to be noted that the vapor-pressure of the solution has been increased by the compression from its normal value p_1 to p_0, that of the solvent, this being required by the Second Law, since a system which at constant temperature is in equilibrium in one way must be in equilibrium in every way.

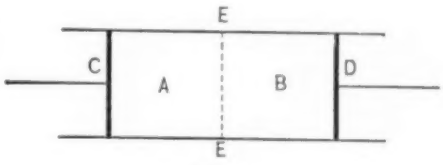


Fig. 1.

Second, suppose that the pressure on piston C is made equal to the vapor pressure p_1 of the solution. Then equilibrium may be brought about by producing on piston D a pressure of $p_1 - P_1$ where P_1 is the osmotic pressure of the solution when standing under its vapor pressure p_1. The vapor pressure of the solvent is reduced by its expansion to that (p_1) of the solution, for the reason before stated.

It is necessary to add that the above definition does not fully coincide with that usually given, osmotic pressure being ordinarily

defined as the total difference in pressure between a solution and solvent separated by a semipermeable wall when each stands under only its own normal vapor-pressure. This total pressure difference (π_1) is evidently the sum of the true osmotic pressure (P_1), as above defined, and the difference in vapor-pressures (p_0 and p_1) of the solvent and solution. That is, $\pi_1 = P_1 + p_0 - p_1$. The quantity π_1 is, however, a pressure that does not correspond to a condition of equilibrium, and could therefore never be directly involved in a reversible process. It is, moreover, the sum of two independent kinds of pressure which vary according to different laws with the temperature and volume, and are differently related to each other in the case of different solvents. Its use seems, therefore, from a physical standpoint, very inadvisable.

3. JUSTIFICATION FOR THE EMPLOYMENT OF LIQUIDS WITH NEGATIVE PRESSURES.

If, as is ordinarily the case, the osmotic pressure of the solution be greater than the vapor-pressure, it would be necessary in the second arrangement for equilibrium described above to subject the solvent to a negative pressure, which in the case of concentrated solutions would have a very high value. It has not, however, hitherto been found practicable even under the most favorable conditions, to make liquids withstand, without breaking apart, a negative pressure much exceeding one atmosphere. On this ground Dieterici has denied the validity of the relation between osmotic pressure and vapor-pressure derived by Noyes and Abbot, for the osmotic column considered by them stands under a negative hydrostatic pressure, and he insists in the words of Nernst that a process or equilibrium-condition to which the Second Law is applied must be "wenigstens prinzipiell experimentell realizirbar." It will be readily seen, however, that this requirement is in reality fulfilled, for though high negative pressures are unattainable, yet, since small ones have been realized, it is always possible to devise a process or arrangement involving the summation of these small pressures, so that the same final result is produced as if the large negative pressure were directly applied. Thus, if one gram of a solvent is to be removed from a solution so concentrated that the requisite negative

pressure on the solvent cannot be realized, a series of solutions of gradually decreasing concentration might be employed for the purpose, each one taking up the gram of solvent from one a little more concentrated and giving it off to one a little more dilute, these intermediate solutions thus undergoing no permanent change and the results being the same as if the solvent were withdrawn directly. Similarly, if the solution is so concentrated that a column of solvent of the height necessary to compensate the osmotic pressure will not hold together, a column of the same height can be built up in sections, provided with semipermeable walls at their tops, and filled with a series of solutions of varying concentrations, increasing as one passes upwards; since such a column is in equilibrium at its top with the concentrated solution and at its bottom with pure solvent standing under its vapor-pressure, its pressure relations must be the same as that of a column of solvent which at its two ends is in equilibrium with the same pressures.

Although the employment of negative pressures is thus justified even in accordance with the assumed requirement that the conditions must be "wenigstens prinzipiell experimentell realizirbar," I feel it is important to add a few words in regard to this dictum, which as interpreted by Dieterici, leads to the absurd result that the thermodynamic relation between osmotic and vapor pressure derived by Noyes and Abbot is exact as long as the osmotic pressure does not exceed the negative pressures which liquids have thus far been found to sustain, but ceases to be valid for slightly higher pressures. The following seems to me a more correct as well as a more definite statement of the matter. What is essential in order that the application of the quantitative form of the Second Law may lead to useful results is the knowledge first of the nature of the intensity-factor within the system which must be compensated in order that the change in question may take place reversibly, and secondly, the knowledge in each special case, of the relation of this intensity-factor to the work involving it, since it is always two quantities of *work* which are brought into relation to each other by the application of the Second Law to an isothermal cyclical process. If these facts in regard to the intensity-factor are known, it is a matter of indifference what external mechanical or other device be employed for its com-

pensation, or whether any such device has been experimentally realized or even imagined. In order that this knowledge in regard to the intensity-factor may rest on a firm experimental basis, it is necessary, to be sure, that some device for its measurement shall have been realized, and that reliable measurements of it shall have been made.

Briefly expressed, the points which I desire to emphasize are that the quantitative form of the Second Law is applicable to every reversible change whether experimentally realizable or not, and that any uncertainty that may attach to the conclusions drawn from it in regard to the intensity-factor involved in the change, arises not from the fact that the Second Law is not applicable to ideal changes, but because the nature of the intensity-factor to be compensated and the form of the condition equation containing it, may not have been correctly understood. For example, van't Hoff's original deduction of the relations of osmotic pressure to vapor-pressure or to freezing-point-lowering is correct, provided his conception of osmotic pressure as the intensity-factor whose external compensation will make the process of addition or separation of the solvent reversible, and his assumption in regard to the volume factor involved, are correct; the validity of his deduction is, however, independent of the experimental realization or mental conception of devices like semipermeable walls which may serve for the compensation of the pressure, except in so far as such devices may actually have been used to demonstrate experimentally the correctness of the conception of osmotic pressure and of its assumed relation to the calculated osmotic work.[1] It should not be forgotten, however, that this requisite knowledge can often be also obtained in an indirect or theoretical manner; and that the degree of certainty which often attaches to it, though not complete, may still be very great. It may, more-

[1] Two further illustrations of the matter under consideration may be taken from Nernst's Theoretische Chemie, 2d edition, pp. 101 and 226. The first case involves the calculation of the work obtainable by the reversible mixing of two different gases, and the author insists that the valididity of the thermo-dynamic relation derived is dependent upon the actual realization of suitable semipermeable walls. In my opinion, however, its validity rests solely on the correctness of the assumption that the partial pressure of each of the two gases in the mixture is the same as if each gas were present alone, an assumption which, to be sure, can be directly tested only through the realization of the

over, be pointed out that the certainty of the conclusions drawn from the cyclical process is not greatly increased by a mere qualitative demonstration of the realizability.

4. Derivation of the Relation by Means of a Cyclical Process.

Consider a cyclical process carried out at constant temperature with the apparatus shown in Fig. 2, in which A is a solution of osmotic pressure P_1 and vapor pressure p_1 separated from the pure solvent B by a semipermeable wall EE and in contact with vapor of the solvent in the space F at the pressure p_1. By means of piston D a pressure $p_1 - P_1$ is exerted on the solvent B just sufficient to prevent it from entering the solution.

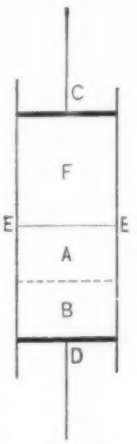


Fig. 2.

Start with one gram of solvent-vapor in the space F and an indefinite small quantity of liquid solvent under the piston D, and carry out the following isothermal process:

1. Slightly increase the pressure on piston C until one gram of the vapor (of volume v_1 at pressure p_1) condenses into the solution and passes out through the wall EE (occupying the volume V' at the pressure $p_1 - P_1$). In this process the solution undergoes no permanent change.

2. Close the openings of the wall EE and allow the one gram of liquid solvent just obtained to contract until the pressure becomes equal to its normal vapor-pressure p_0 and the volume becomes V_0.

3. Reduce slightly the pressure p_0 on the piston until the gram of solvent vaporizes, forming the volume v_0 of vapor at the pressure p_0.

semipermeable walls. The second example concerns the relation between the amounts of work involved in passing isothermally from the liquid to the gaseous condition, in one case at constant vapor pressure, and in the other case along the pressure volume curve predicted by van der Waal's equation. The author does not regard the conclusion reached absolutely binding, for since the realization of the second operation is impossible, he considers that the application to it of the Second Law is questionable. It seems to me, however, that the conclusion is uncertain only to the extent to which the validity of van der Waal's equation is uncertain.

4. Allow the vapor to expand until its pressure becomes p_1 and its volume v_1. The cyclical process is now complete.

The amounts of work W done by the system in the four parts of the process are:

$$W_I = -p_1 v_1 + (p_1 - P_1)V'$$

$$W_{II} = \int_{V'}^{V_0} PdV$$

$$W_{III} = p_0 v_0 - p_0 V_0$$

$$W_{IV} = \int_{v_0}^{v_1} pdv$$

But by the Second Law

$$W_I + W_{II} + W_{III} + W_{IV} = 0,$$

or

$$\int_{v_0}^{v_1} pdv + p_0 v_0 - p_1 v_1 = -\left(\int_{V'}^{V_0} PdV + (p_1 - P_1)V' - p_0 V_0\right)$$

Now since in general:

$$p_b v_b - p_a v_a = \int_{v_a}^{b} pdv + \int_{p_a}^{p_b} vdp$$

it follows that:

$$\int_{p_1}^{p_0} vdp = \int_{(p - P_1)}^{p} VdP. \qquad (2)$$

Equation (2) is the most general and an absolutely exact expression of the relation between vapor pressure and osmotic pressure.

If in the case of the solvent, as is always approximately the case for moderate pressures, the decrease in volume is proportional to the increase of pressure, that is, if

$$-\frac{V - V_0}{V_0} = k_0(P - p_0) \text{ or } V = V_0 - V_0 k_0 (P - p_0) \qquad (3)$$

where k_0, the compression-coefficient of the solvent, is constant, then the second member of (2) can be integrated.

$$\int_{p_1 - P_1}^{p_0} VdP = (V_0 + V_0 k_0 p_0)\int_{p_1 - P_1}^{p_0} dP - \frac{V_0 k_0}{2}\int_{p_1 - P_1}^{p_0} 2PdP$$

whence follows:

$$\int_{p_1}^{p_0} v dp = (P_1 + p_0 - p_1) V_0 \left(1 + \frac{(P_1 + p_0 - p_1) k_0}{2} \right) \qquad (4)$$

This expression can be simplified in form, but not in significance, by substituting for $(P_1 + p_0 - p_1)$ the total pressure-difference π_1. (See above.) It then becomes:

$$\int_{p_1}^{p_0} v dp = \pi_1 V_0 (1 + \tfrac{1}{2} \pi_1 k_0) \qquad (5)$$

5. Derivation of the Relation by Consideration of the Equilibrium of an Osmotic Column.

This same relation can be derived still more simply by a consideration of the conditions of equilibrium which must prevail in an osmotic column. This was attempted in the earlier article by Noyes and Abbot, but as Dieterici has called attention to an error in the sign of the compression term of the formula derived, and as the deduction can be given much more briefly and clearly than was done at that time, I may be permitted to present the matter anew. A vertical tube which is provided [1] with a fixed semipermeable wall near its top opens below into an open vessel. Above the wall is placed an infinitely thin layer of any solution, and the tube and the vessel below it are filled with the pure solvent. The whole is enclosed within a space containing only the vapor of the solvent. Equilibrium would be established by the passage of a small quantity of the solvent through the wall into or out of the solution. If p_1 is the vapor-pressure and P_1 the osmotic pressure of the solution, the solvent just below the wall must then be under the pressure $p_1 - P_1$, in order to compensate the opposite pressures of the solution. On the other hand, the vapor from the solution must be in equilibrium with that from the solvent at the semipermeable wall that is, both must have the same pressure p_1, since otherwise perpetual motion of the second kind would be possible. The pressure on the solvent at its surface in the vessel is, however, its vapor-pressure p_0, and it is evidently possible by the general law of statical pressure to determine, on the one hand the height above the surface at which the pressure of the

[1] For a figure illustrating the arrangement, see Physical Review, 5, 114.

vapor would become p_1, and on the other that at which the pressure of the liquid column of solvent would become $p_1 - P_1$. Since these two heights must be equal, a relation between osmotic and vapor pressures is thereby obtained. Namely, since the difference in pressure at any two heights is equal to the weight of the intervening column of vapor or liquid solvent, we have the relations

$$\delta dh = -dp \text{ and } \Delta dh = -dP$$

where δ and Δ are the densities of the vapor and liquid respectively, and dp and dP are their respective changes in pressure; or:

$$dh = -vdp \text{ and } dh = -VdP$$

where v and V are the (variable) specific volumes of the two. If the height of the wall above the surface is h_1, we have

$$\int_0^{h_1} dh = h_1 = -\int_{p_0}^{p_1} vdp = -\int_{p_0}^{p_1 - P_1} VdP$$

which is a relation identical with equation (2) derived by the cyclical process. From it, by substitution of the compression-coefficient (see equation (3)) equations (4) and (5) are obtained as before.

6. Discussion of the Derived Relation.

These equations contain, as Dieterici points out, a quantity which can not be directly experimentally determined, namely k_0, which is the mean compression-coefficient of the solvent between the pressures p_0 and $p_1 - P_1$, the latter of which is in general negative. This is true, and the fact does introduce, it must be admitted, a certain degree of uncertainty into practical applications of the formula: when it is considered, however, that the measurements of various investigators have shown that the compression-coefficient varies but slightly even for considerable variations of positive pressures,[1] it seems highly probable that the value for moderate negative pressures will not be greatly different from that for low positive pressures. More-

[1] Compare the summary of values in Landolt and Börnstein's Tabellen, p. 265. One or two examples may be here cited: alcohol at 28° has a coefficient of 86×10^{-6} between 150 and 200 atmospheres, and of 81×10^{-6} between 150 and 400 atmospheres; for ether at 13° the value is 168×10^{-6} between 8.4 and 13.9 atmospheres, and 165 between 8.4 and 36.5 atmospheres.

over, the error in the compression coefficient would have to be very large indeed, in order to produce an appreciable effect upon the calculated value of the osmotic pressure; for the whole term $\frac{1}{2}\pi k_0$ has a relatively small value, even for quite concentrated solutions; thus an ether solution must have an osmotic pressure of about 120 atmospheres, an alcohol solution one of 250 atmospheres, and a water solution one of 400 atmospheres, before this term amounts to one per cent.

It is to be especially pointed out that the volume-factor which occurs in the above equations is the volume of one gram of the *solvent* under its normal vapor pressure corrected by a term (relatively small even for fairly concentrated solutions) representing the effect of its compression. But the factor which has hitherto been used by Dieterici and others in calculating the osmotic pressure from the integral $\int_{p_0}^{p_0} pdv$ is the *change in the volume of the solution* that occurs when one gram of solvent is added to an indefinitely large quantity of the solution. These two volume-factors have, however, essentially different values in the case of even moderately concentrated solutions, as is shown by the following table which contains the results for sulphuric acid solutions.[1] The first column contains the concentration in mols per liter; the second, the value of the integral $\int_{p_1}^{p_0} vdp$ at 0°, as determined experimentally by Dieterici, this being approximately equal to the osmotic pressure in kilograms per square centimeter; the third, the volume change V_1, when one gram of water is added to an infinite amount of the solutions, as calculated by Dieterici; the fourth, the value of $V_0(1 + \frac{1}{2}\pi_1 k_0)$ occurring in equation (5) calculated by me assuming the compression coefficient (k_0) of water at 0° to be 0.000050; and the fifth the percentage differences between the values in the two preceding columns.[2]

[1] For data in regard to the volume change of other solutions see Arrhenius, Zeitschr. für physik. Chem., 10, 93.

[2] Owing to the error in sign previously referred to, the fourth column of the corresponding table published in my previous article (28, 223) contained the values of $V_0(1 - \frac{1}{2}\pi_1 k_0)$, and the percentage differences between those values and Dieterici's were much smaller than the correct ones which are given in this table.

Mols per Liter.	$\int_{p_1}^{p_0} v dp$	V_1	$V_0(1+\frac{1}{2}\tau_1 k_0)$	Percentage differences.
0.607	23.66	0.999	1.000	0.1
1.040	47.32	0.997	1.001	0.4
1.903	97.07	0.990	1.002	1.2
2.681	164.6	0.989	1.004	1.5
3.792	297.8	0.979	1.007	2.8
4.908	456.8	0.970	1.011	4.1
5.598	575.8	0.967	1.014	4.7
8.164	1028.0	0.959	1.025	6.6
9.43	1301.0	0.958	1.032	7.4
11.69	1727.0	0.957	1.042	8.5
16.19	2693.0	0.947	1.064	11.7
22.18	4170.0	0.930	1.096	16.6

Since the values of the two volume-factors are essentially different from each other, it is evident that the osmotic pressures that would be obtained by dividing the integral by them would also differ, and that therefore one of the two methods of calculation must be inexact, or must refer to the osmotic pressure of the solution under a different condition. In view of the simplicity of the deductions given above, both by means of the cyclical process and the osmotic column, the question might well be considered decided in favor of the result thereby reached, at any rate until some real error is pointed out in my method of treatment. However, since Dieterici in his last article still maintains the correctness of the volume-factor representing the change of volume of the solution, it seems desirable to show by a consideration of a cyclical process involving that factor, the source of the error. A process of this kind has been employed for deriving the relation between osmotic and vapor pressures by previous authors, but they have intentionally left out of consideration the work done in compressing and dilating the solution and the variation of the osmotic pressure with the compression of the solution. This must not be done, however, if theoretically exact relations are to be derived.

7. Derivation of the Relation by Means of a Second Cyclical Process.

We will therefore consider the following completely reversible cyclical process, starting with one gram of solvent vapor of volume

v_1 and an indefinite large quantity of solution of volume $\omega - V_1$, these being in equilibrium with each other at the pressure p_1.

1. Condense the solvent vapor into the solution. Let its volume become thereby ω and its vapor pressure $p_1 + dp_1$. V_1 then represents the volume-factor above discussed, *i. e.*, the change in volume of the solution when one gram of solvent is added to an infinite quantity of it. The average pressure during these volume changes is $p_1 + \frac{1}{2}dp_1$.

2. Compress the solution until its pressure becomes such that it is in equilibrium with the pure solvent under its own vapor-pressure p_0, when placed in communication with it through a semipermeable wall. The external pressure must then become $p_0 + P_1' - dP_1'$, if $P_1' - dP_1'$ is the true osmotic pressure of the compressed solution. If the compression-coefficient of the solution is constant and equal to $k_1 + dk_1$, its volume after the compression is $\omega - \omega(k_1 + dk_1)(P_1' + p_0 - p_1 - dP_1' - dp_1)$. The average pressure under which this compression takes place is $\frac{1}{2}(P_1' + p_0 + p_1 - dP_1' + dp_1)$.

3. Force one gram of solvent out of the solution through the semipermeable wall. Let the volume of the pure solvent so obtained under the pressure p_0 be V_0. The external pressure on the solution becomes $p_0 + P_1'$ if the true osmotic pressure of the resulting solution is P_1'. If its compression-coefficient is constant and equal to k_1 its volume is $\omega - V_1 - (\omega - V_1)k_1(P_1' + p_0 - p_1)$. The average pressure under which the change of volume of the solution takes place is $P_1' + p_0 - \frac{1}{2}dP_1'$.

4. Expand the solution till its pressure becomes p_1. Its volume is then $\omega - V_1$; and it has returned to its initial condition. The average pressure under which this expansion takes place is $\frac{1}{2}(P_1' + p_0 + p_1)$.

5. Vaporize the one gram of solvent at the pressure p_0 (its volume then becoming v_0) and allow the vapor to expand till the pressure becomes p_1 and the volume v_1.

The cyclical process is now complete.

In order to calculate the work corresponding to the volume-changes of the solution in this process, the change of volume is to be multiplied by the average pressure under which it takes place, it being thereby assumed that the compression-coefficient is indepen-

dent of the pressure. If the amounts of work done on the solution in the different parts of the process are thus calculated and added to the work done on the solvent, and their sum placed equal to zero, small quantities of the second order being neglected (noting however that those of the form $\omega d(-)$ are finite since ω is indefinitely large), the following equation is obtained:

$$\int_{P_1}^{P_0} v dp = (P_1' + p_0 - p_1)[V_1\{1 - \tfrac{1}{2}k_1(P_1' + p_0 - p_1)\} + \omega k_1 dp_1 - \tfrac{1}{2}\omega(P_1' + p_0 - p_1)dk_1].$$

In this equation the terms containing ω can be given a more readily intelligible form, if we designate the concentration of the solution of volume $\omega - V_1$ by c_1 and that of volume ω containing one gram more of solvent by $c_1 - dc_1$ and note that:

$$c_1 : c_1 - dc_1 = \omega : \omega - V_1$$

and therefore:

$$\omega = V_1 \frac{c_1}{dc_1}.$$

Substituting this value of ω, the last equation then becomes:

$$\int_{P_1}^{P_0} v dp = (P_1' + p_0 - p_1)V_1 \left(1 - \tfrac{1}{2}k_1(P_1' + p_0 - p_1) + k_1 c_1 \frac{dp_1}{dc_1} - \tfrac{1}{2}c_1(P_1' + p_0 - p_1)\frac{dk_1}{dc_1}\right). \quad (6)$$

In the special case where the vapor-pressure difference can be neglected in comparison with the osmotic pressure, this becomes:

$$\int_{P_1}^{P_0} v dp = P_1' V_1 \left(1 - \tfrac{1}{2}k_1 P_1' - \tfrac{1}{2}c_1 \frac{dk_1}{dc_1} P_1'\right) \quad (7)$$

8. Comparison of the Relations Derived by the Two Cyclical Processes.

Equation (6) like equation (4) is rigidly exact, except in so far as error is introduced by the assumption that the compression-coefficient is independent of the pressure. The two equations are seen to differ with respect to all the quantities occurring in them except

the vapor-pressures. Equation (6) is much more complex than equation (4), and has, moreover, one other characteristic which greatly diminishes its theoretical and practical value. Namely, it will be noted that it gives a relation between the osmotic pressure P_1' of a compressed solution when under the external pressure $(P_1' + p_0)$ and the vapor pressure of the same solution when under its own normal vapor pressure p_1 and that of the pure solvent also under its own normal vapor pressure p_0. In other words, it brings into relation the two kinds of properties measured under different external conditions of pressure. This fact alone is sufficient to make the result of little importance from a theoretical standpoint, and to show the unfitness of this ordinarily employed cyclical process as a means of deriving the exact value of the osmotic pressure P_1 at ordinary pressure, which is involved in almost all applications; such, for example, as the calculation of electromotive force. It is true, to be sure, that this osmotic pressure P_1' is the pressure that has been thus far experimentally determined by Pfeffer and others, no method having been yet found for producing the external negative pressure requisite in determining P_1 or for measuring directly the pressure on the semi-permeable wall itself.

It may be further pointed out that most of the other quantities occurring in equations (6) and (7) $\left(\text{namely } V_1, k_1, \text{ and } \frac{dk_1}{dc_1}\right)$ vary with the concentration and nature of the solute, and must, therefore, be determined for each solution to which the formula is applied, while the corresponding quantities (V_0 and k_0) in equation (4) refer to the solvent, and hold true for any solution whatever in that solvent.

9. Conclusion.

It has been shown in this article by the consideration both of a new cyclical process and of the equilibrium-conditions of an osmotic column that the exact thermodynamic relation between the osmotic pressure (P_1) and vapor-pressure (p_1) of a solution is given by the equation:

$$\int_{p_1}^{p_0} v\,dp = (P_1 + p_0 - p_1)V_0[1 + \tfrac{1}{2}(P_1 + p_0 - p_1)k_0]$$

in which p_0, V_0, and k_0 are the vapor-pressure, specific volume, and compression-coefficient respectively of the liquid solvent. The only simplifying assumption made in the derivation of this equation was that k_0 could be regarded as constant between the pressures p_0 and $p_1 - P_1$, the latter pressure being in general negative. Since the value of the term containing the compression-coefficient (k_0) is very small, even for fairly high concentrations, the approximate osmotic pressure of a solution is correctly calculated by dividing the vapor-pressure integral by the ordinary specific volume of the solvent, and not, as has been generally done heretofore, by dividing it by the volume-change of the solution. It has also been shown that the derivation by the cyclical process heretofore employed, when rigidly carried out, is far less simple in character, and gives as a result a more complex relation, which involves, moreover, the osmotic pressure of the compressed solution, not that of the solution under its own pressure, and a number of other quantities which have to be determined specifically for each solution to which the equation is applied, instead of determining them for the solvent, once for all.

POSTSCRIPT.

The beautiful investigation of Worthington on "The Mechanical Stretching of Liquids: an Experimental Determination of the Volume-Extensibility of Ethyl Alcohol" (Philos. Trans. of the Royal Society, 183, pp. 355–370. 1892) first came to my attention while reading the proof of the foregoing article. This investigation has demonstrated the possibility not only of realizing high negative pressures (up to 17 atmospheres), but also even of measuring the dilatation-coefficient with a fair degree of accuracy. It has also proved, in the words of the author, that "in the neighborhood of zero pressure the absolute coefficient of volume elasticity of alcohol is the same for extension as for compression, and, so far as the observations show, is constant between a pressure of + 12 and − 17 atmospheres." Any hypothetical element which may have been thought to be involved in the deduction given in the preceding article by reason of the employment of negative pressures is, therefore, to a great extent, removed; and in applications of the

derived equation the value of the compression-coefficient measured for small positive pressures can be generally employed with a much greater degree of confidence, in view of the demonstrated constancy of the coefficient for positive and negative pressures in the case of one liquid.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS., November, 1900.

DEPENDENCE OF THE MODULUS OF TORSION ON TENSION.

BY J. R. BENTON.

IT is to be expected from the theory of elasticity that the coefficients of elasticity of a body should vary as it is deformed. If X_x, Y_y, Z_z, denote the normal components of stress on planes perpendicular respectively to the X, Y and Z axes of coördinates; Y_z, Z_x, X_y, the tangential components on the same planes; and x_x, y_y, z_z, y_z, z_x, x_y, the corresponding components of strain; then the relation between stress and strain can be written in the general form

$$\begin{aligned} X_x &= f_1(x_x, y_y, z_z, y_z, z_x, x_y) \\ Y_y &= f_2(x_x, y_y, z_z, y_z, z_x, x_y) \end{aligned} \qquad (1)$$

with similar formulas for the other components of stress. f_1, f_2 denote unknown functions. If these functions are developed according to Maclaurin's formula, we have

$$\begin{aligned} X_x = {} & c_{11}x_x + c_{12}y_y + c_{13}z_z + c_{14}y_z + c_{15}z_x + c_{16}x_y + d_{11}x_x^2 + d_{12}y_y^2 \\ & + \cdots + d_{17}\cdot 2x_xy_y + \cdots + e_{11}x_x^3 + \cdots \qquad (2) \\ Y_y = {} & c_{21}x_x + c_{22}y_y + \cdots + d_{21}x_x^2 + \cdots \end{aligned}$$

with similar equations for Z_z, Y_z, Z_x, and X_y; the c's, d's, e's, etc., denote constants. The constant term $f(0)$, which is usually placed at the beginning of Maclaurin's series, is omitted, because it is assumed that the stress disappears when all the strains disappear. The strains are exceedingly small quantities in comparison with the stresses; it is therefore customary to neglect their squares and products, and to break off the series (2) after the sixth term. The resulting equations form the basis of the ordinary theory of elasticity; they make the strains proportional to the stresses, the coefficients remaining unchanged as the strains increase indefinitely.

If, however, the strains are so large or so accurately known that their squares cannot be neglected, then the series (2) must include the terms of the second degree. This gives

$$\begin{aligned} X_x = {} & (c_{11} + d_{11}x_x + d_{17}y_y + d_{18}z_z + d_{19}y_z + d_{110}z_x + d_{111}x_y)x_x \\ & + (c_{12} + d_{12}y_y + d_{17}x_x + d_{112}z_z + d_{113}y_z + d_{114}z_x + d_{115}x_y)y_y \qquad (3) \\ & + (c_{13} + \cdots)z_z + (\cdots)y_z + (\cdots)z_x + (\cdots)x_y \end{aligned}$$

and similar expressions for Y_y, Z_z, Y_x, Z_x, X_y. It is evident that the proportionality between stress and strain no longer subsists. In place of the ordinary coefficients of elasticity the c's of our formulas, we have the expressions in ( ) in (3); the latter fill exactly the place of the c's in the ordinary theory of elasticity, and can therefore be regarded as new coefficients of elasticity which, however, depend on the strain and therefore vary as the body is deformed.

The theory of the dependence of the coefficients of elasticity on the deformation in the case of a homogeneous, isotropic body, has been worked out in two forms. Voigt,[1] assuming that the strains can be neglected in comparison to the size of the body, deduced formulas which represent the behavior of a homogeneous, isotropic body by means of three new constants of elasticity in addition to the two of the usual theory. Jos. Finger,[2] independently of Voigt, investigated the same problem, but did not neglect the strain in comparison to the size of the body. He arrived at formulas which contain four new constants, besides the two of the old theory of elasticity.

Experiments have verified the theory, and shown that the coefficients of elasticity depend in a slight degree on the deformation, even far within the elastic limit, and that, therefore, the strain is not exactly proportional to the stress. This seems to have been known as early as 1705, when James Bernouilli[3] wrote : " Des fibres homogènes de même longueur et largeur, mais chargées de differents poids, ne s'étendent ni se compriment pas proportionellement à ces poids ;

[1] W. Voigt, Wiedemann's Annalen, 52, p. 536, 1894.

[2] Jos. Finger, Wiener Berichte, Band 103, Abth. IIa, 163, 1894.

[3] J. Bernouilli, Gesammelte Werke, Band II., 976–989. Geneva, 1744. Quoted in Todhunter and Pearson, History of the Theory of Elasticity and of the Strength of Materials. Cambridge, 1886.

mais l'extension ou la compression causée par le plus grand poids, est à l'extension ou la compression causée par le plus petit, en moindre raison que ce poids-là n'est à celui-ci." During the present century numerous experiments on the elasticity of extension, especially those of Eaton Hodgkinson [1] on structural iron and steel, and those of J. O. Thomson [2] on wires of various metals, have shown that for most substances the increase of length is not exactly proportional to the stretching force.

On the other hand, Voigt's careful experiments on the torsion and flexure of crystals failed to show any deviation from proportionality, although he carried the deformations almost to the breaking point. This result, however, follows from Voigt's development of the theory of the dependence of elastic constants on deformation; according to which the deviation from proportionality appears in a term of the first order in the case of extension, but in a term of the second order in the cases of torsion or flexure.

The modulus of torsion, then, according to theory, does not vary appreciably with the torsion; it does vary appreciably, however, with the extension in the direction of the axis of torsion. The theory enables us to express the modulus of torsion in terms of the tension in the direction of the axis. If m denote the modulus, P the tension, and α, β, γ be constants, then

$$m = \alpha + \beta P + \gamma P^2. \qquad (4)$$

This formula follows from the theory, both under Voigt's assumption that the strains can be neglected with respect to the dimensions of the body, and under Finger's assumption that they cannot; only the constants have a somewhat different meaning in the two cases. In both cases α is the ordinary modulus of torsion, when no tension is present; β and γ are rather complicated functions of the five (according to Voigt) or six (according to Finger) elastic constants of the substance in question. The mathematical details of the deduction of the above formula may be left out here.[3]

[1] E. Hodgkinson, Report of the Commission appointed to enquire into the Application of Iron to Railway Structures. London, 1849, pp. 47–67.

[2] J. O. Thomson, Wiedemann's Annalen, 44, p. 555, 1891.

[3] The mathematical deduction will be found in the Göttingen Dissertation—"Abhängigkeit der specifischen Torsionswiderstandes einiger Metalldrähte von der Spannung." J. R. Benton, 1900.

The results of the experiments described below were found to agree well with this formula.

The experimental investigation of the changes of the modulus of torsion with the tension was suggested by Lord Kelvin[1] in 1878, but no experiments were made until those of Cantone and Michelucci[2] in 1897. They used wires of iron, copper, brass, nickel, and silver, which hung vertically, and were twisted by means of horizontal pulleys at the bottom of the wires. The wires were repeatedly loaded and unloaded, and the change of the angle of torsion at each change of load observed. Experiments were made with different values for the original angle of torsion. It was found that an increase of tension was accompanied by an increase of the angle of torsion, therefore by a decrease of the modulus of torsion. Nickel, however, behaved exactly oppositely to the other wires. Cantone and Michelucci did not attempt to formulate a law for the variation of the modulus of torsion; and they seem to have left out of account the influence of the change of shape of the wire when loaded; the fact of its becoming longer and thinner would change its amount of torsion, even if the modulus remained the same.

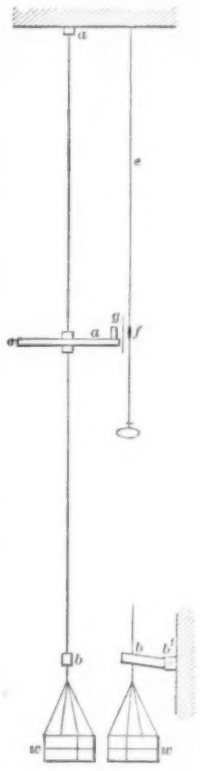


Fig. 1.

It was suggested to me by Professor Voigt to investigate the subject more exhaustively, using more exact methods. The experiments described here were made in his laboratory in the University of Göttingen.

The method used to determine the modulus of torsion under the various tensions consisted in comparing, by means of the method of coincidences, the period of torsional vibration of a disk fastened to the wire, with the period of a pendulum swinging in front of the disk.

[1] Lord Kelvin, Article "Elasticity" in the Encyclopedia Britannica.

[2] M. Cantone and E. Michelucci, Rendiconti della reale Accademia dei Lincei (5), 6, pp. 191–198, 1897.

A wire about three meters long was fastened firmly to the ceiling of the room by means of the clamp *a* (Fig. 1). The lower end of the wire was clamped in the end of the arm *b*, about 8 cm. long, which was free to turn about a horizontal axis *b'*. In this way the extension of the wire was not interfered with, while sidewise motion of the lower end was prevented, and the torsion was limited to the the space between *a* and *b*. Exactly in the middle of this space a brass disk *c*, 18 cm. in diameter, was fastened. For wires of different thicknesses I used disks of different weights, ranging from 0.7 kg. to 2.2 kg.

The disk made torsional vibrations. The tension of the wire was produced by means of weights hung on below the arm *b*. I took special pains to see that the points at which the wire was clamped in *a* and *b* did not change when the wire contracted as a result of the tension.

On the edge of the disk *c* a little piece of sheet iron *d*, about 1 cm. high, and bent to a right angle, was set up vertically. The two sides of the angle made equal angles with the radius of the disk, so that the resistance of the air was the same when the disk swung in one direction as when it swung in the other; one side was painted white so as to be distinctly visible, the other side was blackened. The white side was strongly illuminated (when the disk was in its position of rest) by the light of a gas burner concentrated by means of a lens. The whole angle *d* was movable along the edge of the disk, and could be set where it was wanted.

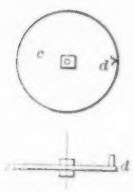


Fig. 2.

In front of the disk hung a pendulum, consisting of a lens-shaped piece of lead hung on a fine steel wire. At the same height as the angle *d*, this fine wire carried a little black cylinder *f*, of such a diameter that it just covered the white side of the angle *d*, when looked at in the horizontal line passing through the pendulum wire and the wire to be investigated. Between the disk and the pendulum wire was a black screen *g*, with a rectangular hole in it of just the same breadth as the diameter of the cylinder. When the pendulum and disk were at rest, the centers of the wire, the white half of the angle *d*, the hole in the screen, and the black cylinder

were all in the same straight line. A telescope was set up in that line at a distance of about three meters from the wire, so that the vibrations of the pendulum and disk could be observed from a distance.

The length of the pendulum was so adjusted that its period T was nearly equal to that of the torsional vibrations of the disk (or else $2T$ or $3T$ were made nearly equal to the period of the disk, in cases where it swung more slowly). The coincidences were observed through the telescope; a coincidence taking place when the black cylinder f passed the hole in the screen g at the same time (and in the same direction) as the illuminated side of the angle d, and hid the latter from view. A perfect coincidence rarely took place; the cylinder f usually left part of the bright angle uncovered; at one swing the bright part appeared on one side, at the next on the other side of the cylinder. It was possible, however, by watching a few swings before and after the coincidence, to estimate in tenths of a pendulum swing, the true time of the coincidence.

If θ is the interval between two successive coincidences, the period S of the disk is given by

$$S = \frac{\frac{\theta}{T}}{\frac{\theta}{T} \pm 1} \cdot T = \frac{\theta}{\theta \pm T} \cdot T \qquad (5)$$

It will be seen that an error of the first order in T appears only in the second order in $\frac{\theta}{\theta \pm T}$, so that the period of the disk *in terms of* T can be determined very accurately when T is known only approximately. T was determined by measuring three times with a stop-watch (reading to 0.2 second) the time of 100 swings of the pendulum. The error of T may be taken as $\pm$ 0.002 second; that of $\frac{\theta}{\theta \pm T}$ is then $\frac{\mp 0.002\,\theta}{(\theta \pm T)^2}$. If T remains the same for the different determinations of S under different tensions of the same wire, the values of S can be compared very accurately without the necessity of a very accurate determination of T.

The length of the pendulum was so adjusted that a coincidence occurred about every two and one-half minutes.

The pendulum lost its motion very slowly, and an amplitude of more than one and one-half degrees was never given to it, so that its period could be regarded as absolutely constant. The variations of its period during a series of observations must have amounted to less than a thousandth of one per cent. The period of the disk, on the contrary, varied considerably with the amplitude for most wires. The causes of this variation—resistance of the air, internal friction of the wire, adiabatic temperature changes due to the deformation, and a possible variation of the elasticity with the angle of torsion—were so complicated that it did not seem practicable to investigate them separately ; the dependance of the period on the amplitude was therefore determined empirically. This was done by making a long series of observations as the amplitude died away. It was found that the time θ between two coincidences could be represented with very close approximation as a linear function of the amplitude A.

$$\theta = \theta_0 + bA. \tag{6}$$

This empirical result holds true up to about 50°, but the amplitudes in my experiments never exceeded 35°, and usually lay between 10° and 20°.

Thermometers were placed at the top, the middle, and the bottom of the wire, and the temperature was measured by them, and kept constant within about 2° by opening or closing a door which led to a cooler room. Sudden currents of air were avoided.

Each wire, soon after it was hung from the ceiling, was heated twice to dull redness by means of a Bunsen burner, under the greatest safe load, in order to make it hang straight. After it had cooled, it was loaded to somewhat more than the greatest load to be used in the experiments, and left hanging free for one to two days. Then it was clamped at the bottom, and the observations were begun.

In order to determine the period of the disk when the wire was under any given tension, a series of ten coincidences was observed. For some wires, however, the vibration died down so rapidly that I had to be satisfied with seven coincidences ; while for some others the amplitude remained so nearly constant that the observations had to be continued for 15 or 20 coincidences, in order to determine the effect of the amplitude on the period. At each coincidence the

time was determined by means of a stop-watch (the difference of the observed, approximate coincidence from the true coincidence being estimated), and the amplitude of the disk was observed by means of a scale of angles marked on its edge. To determine the period at the amplitude zero, equations of the form (6) were set up, and the constants θ_0 and b determined from them by the method of least squares. θ_0 is the time between two coincidences at the amplitude zero; from it the period S of the disk was calculated, in terms of the period T of the pendulum.

The modulus of torsion m is given by the equation

$$m = \frac{2\pi K}{S^2} \cdot \frac{l}{r^4} \tag{7}$$

where K is the moment of inertia of the disk and wire, l is the length, and r the radius of the wire. To use this formula an accurate knowledge of T and K is necessary. But inasmuch as I was investigating only the *changes* of the modulus of torsion, and not its absolute value, I simply compared m with m_s, the modulus of torsion under a standard load; T and K, of course, remained the same. This gives

$$\frac{m}{m_s} = \frac{S_s^2 l r_s^4}{S^2 l_s r^4}, \quad \text{or} \quad m = \left(\frac{S_s}{S}\right)^2 \cdot \left(\frac{l}{l_s}\right) \cdot \left(\frac{r_s}{r}\right)^4 \cdot m_s. \tag{8}$$

The changes of l and r are those due to the linear extension and the lateral contraction of the wire when loaded. $\frac{l}{l_s}$ was determined by direct observation. A needle was fastened to the lower end of the wire and the distance through which it moved as the wire was loaded was read by means of a microscope with micrometer screw-head. The quantity $\frac{r_s}{r}$ was also determined directly, the contraction of the wire being measured by means of an interference apparatus. For a detailed account of the method used, see Article: "Determination of Poisson's Ratio by Means of an Interference Apparatus," PHYS. REV. $\frac{l}{l_s}$ and $\frac{r_s}{r}$ were very slightly different from unity, so that it was possible to assume Hooke's law, and regard the deformation as proportional to the tension.

Each wire was observed under five different tensions; three series of observations were made; first one beginning with the smallest tension, and increasing the tension continuously to the greatest; then one beginning with the greatest and decreasing continuously to the smallest; finally one beginning with the smallest again. Thus fifteen determinations were made with each wire, each determination consisting in the observation of a series of coincidences.

The results obtained are given in the tables below. The first column gives the substance of the wire, the second column its diameter in millimeters, the third the loads in kilograms, the fourth the tensions produced by them in kilograms per square millimeter; the fifth and sixth columns give the modulus of torsion in terms of the modulus of torsion under (an assumed) standard tension, the fifth as observed and the sixth as calculated from the formula (4). The seventh column gives the approximate absolute value of the modulus of torsion.

Substance.	Diameter.	Load.	Tension.	Modulus of Torsion.		
				Observed.	Calculat'd.	Absolute value.
Steel	0.685	2	5.43	1.00364	1.00362	7.523×10^{11}
		5	13.57	1.00219	1.00243	
		10	27.14	1.00000	0.99979	
		15	40.71	0.99608	0.99633	
		18	48.85	0.99386	0.99386	
Steel	1.000	10	12.73	1.00242	1.00247	8.077×10^{11}
		15	19.10	1.00120	1.00119	
		20	25.46	1.00000	0.99997	
		24.7	31.45	0.99897	0.99886	
		29.4	37.43	0.99772	0.99780	
Steel	1.405	10	6.45	1.00134	1.00142	8.030×10^{11}
		15	9.68	1.00059	1.00059	
		20	12.91	1.00000	0.99993	
		24.7	15.95	0.99949	0.99940	
		29.4	18.98	0.99894	0.99902	
Iron	0.883	3	4.89	1.00173	1.00213	5.671×10^{11}
		6	9.84	1.00185	1.00131	
		9	14.68	1.00000	0.99983	
		12	19.57	0.99771	0.99820	
		15	24.47	0.99600	0.99589	

Substance.	Diameter.	Load.	Tension.	Modulus of Torsion.		
				Observed.	Calculat'd.	Absolute value.
Iron	1.190	5	4.50	1.00126	1.00126	6.372×10^{11}
		10	9.00	1.00058	1.00065	
		15	13.49	1.00000	0.99998	
		20	17.99	0.99932	0.99928	
		25	22.49	0.99859	0.99856	
Iron	1.578	10	5.00	1.00088	1.00090	6.732×10^{11}
		15	7.51	1.00063	1.00057	
		20	10.01	1.00000	1.00026	
		25	12.51	1.00042	0.99996	
		30	15.02	0.99963	0.99971	
Copper	0.947	2	2.84	1.00490	1.00501	7.057×10^{11}
		4	5.68	1.00400	1.00340	
		6	8.51	1.00000	1.00060	
		8	11.35	0.99659	0.99651	
		10	14.19	0.99058	0.99072	
Copper	1.390	2	1.32	0.99997	1.00000	6.139×10^{11}
		5	3.30	1.00027	1.00030	
		10	66.59	1.00000	0.99986	
		15	9.89	0.99803	0.99824	
		18	11.86	0.99680	0.99669	
Copper	1.728	10	4.24	1.00252	1.00122	5.342×10^{11}
		15	6.37	1.00077	1.00120	
		20	8.49	1.00000	1.00050	
		25	10.61	0.99753	0.99891	
		30	12.73	0.99760	0.99651	
Nickel	0.818	2	3.72	0.99581	0.99504	8.952×10^{11}
		5	9.29	0.99585	0.99610	
		10	18.58	1.00000	1.00000	
		15	27.87	1.00735	1.00644	
		18	33.44	1.01054	1.01130	
Nickel	0.963	2	2.74	0.99513	0.99489	9.026×10^{11}
		5	6.86	0.99649	0.99685	
		10	13.73	1.00000	1.00004	
		15	20.59	1.00352	1.00309	
		18	24.71	1.00463	1.00487	
Nickel	1.490	10	5.73	0.99734	0.99688	5.518×10^{11}
		15	8.60	0.99822	0.99862	
		20	11.47	1.00000	1.00014	
		25	14.34	1.00134	1.00147	
		30	17.20	1.00278	1.00258	
Brass	0.787	2	4.11	1.00384	1.00417	3.688×10^{11}
		5	10.28	1.00169	1.00173	
		8	16.44	1.00000	0.99917	
		11	22.61	0.99573	0.99641	
		14	28.77	0.99353	0.99343	

Substance.	Diameter.	Load.	Tension.	Modulus of Torsion.		
				Observed.	Calculat'd.	Absolute value.
Brass	0.943	2	2.87	1.00110	1.00111	3.855×10^{11}
		4	5.74	0.99980	1.00050	
		6	8.60	1.00000	0.99993	
		8	11.47	0.99948	0.99933	
		10	14.34	0.99852	0.99870	
Brass	1.498	7	3.97	1.00057	1.00058	3.804×10^{11}
		10	5.67	1.00037	1.00038	
		15	8.51	1.00000	0.99999	
		20	11.35	0.99951	0.99953	
		22.8	12.94	0.99926	0.99925	
Nickelin	0.795	2	4.03	1.00946	1.00943	5.921×10^{11}
		5	10.07	1.00559	1.00571	
		10	20.14	1.00000	0.99987	
		15	30.21	0.99444	0.99449	
		18	36.24	0.99146	0.99147	
Nickelin	0.978	2	2.66	1.00410	1.00426	6.166×10^{11}
		5	6.65	1.00300	1.00270	
		10	13.31	1.00000	0.99998	
		15	19.96	0.99677	0.99711	
		18	23.96	0.99554	0.99531	
Nickelin	1.397	10	6.52	1.00144	1.00140	6.573×10^{11}
		15	9.78	1.00063	1.00071	
		20	13.05	1.00000	1.00011	
		25	16.31	0.99989	0.99961	
		30	19.57	0.99908	0.99921	
German silver	1.005	2	2.53	1.00417	1.00417	7.451×10^{11}
		5	6.33	1.00271	1.00270	
		10	12.67	1.00000	0.99999	
		15	19.00	0.99690	0.99694	
		18	22.81	0.99500	0.99496	
German silver	1.592	10	5.02	1.000526	1.000522	6.584×10^{11}
		15	7.53	1.000232	1.000245	
		20	10.05	1.000000	0.999986	
		25	12.56	0.999736	0.999742	
		30	15.07	0.999516	0.999515	
Brass with large percentage of copper.	0.996	2	2.57	1.00318	1.00320	3.804×10^{11}
		5	6.42	1.00280	1.00310	
		10	12.84	1.00000	0.99943	
		15	19.26	0.99126	0.99140	
		18	23.11	0.98445	0.98460	
Bronze	0.902	2	2.82	1.00451	1.00437	9.077×10^{11}
		4	5.65	1.00158	1.00196	
		6	8.47	1.00000	0.99972	
		8	11.30	0.99762	0.99762	
		10	14.12	0.99565	0.99570	

From the above observations the following values of β and γ in the formula $m = \alpha + \beta P + \gamma P^2$ result. 1 kg. per square mm. is taken as the unit of tension.

Substance.	Diameter of Wire.	Modulus of Torsion.	
		Observed.	Calculated.
Steel	0.685	−0.000104	−0.00000222
	1.000	−0.000218	+0.00000061
	1.405	−0.000367	+0.00000698
Iron	0.883	+0.000011	−0.0000105
	1.190	−0.000127	−0.0000007
	1.595	−0.000149	+0.0000013
Copper.	0.947	+0.000168	−0.0000843
	1.390	+0.000406	−0.0000545
	1.792	+0.001042	−0.0000895
Nickel	0.818	+0.000022	+0.0000135
	0.963	+0.000493	−0.0000013
	1.490	+0.000788	−0.0000126
Brass	0.787	−0.000339	−0.00000275
	0.943	−0.000360	−0.00000099
	1.498	−0.000082	−0.00000039
Nickelin	0.795	−0.000639	+0.00000220
	0.978	−0.000371	−0.00000175
	1.397	−0.000285	+0.00000449
German silver	1.005	−0.000305	−0.00000407
	1.592	−0.000126	+0.00000131
Brass with large percentage of copper.	0.996	+0.000439	−0.0000524
Bronze.	0.902	−0.001961	+0.0000397

These results differ for wires of different thickness of the same material. This must be due to the inhomogeneity of the surface, which would have a greater effect on thin wires than on thick ones. It is well known that working the surface of metals has a marked effect on their elasticity. The drawing of a wire must affect its surface; besides which the surface absorbs gases and vapors, and it is the seat of chemical action, the products of which it is not easy to entirely remove. The wires of different sizes give quite different values for the modulus of torsion, so that it is not surprising that the values of β and γ also vary.

The values found then, cannot be taken as universal constants for the substance in question, but must be regarded simply as constants which give the behavior of particular wires. As the thickness of the wires is increased, β and γ approach values which must be the universal ones for the substance in question. So that by a consideration of the change of β and γ with increasing diameter of the wire, it is possible to obtain a certain amount of knowledge about them. They may be either positive or negative; β probably never exceeds 0.001 m_s and is generally much less, γ is less than 0.00005 m_s. The algebraic sign of β and γ for the different substances is as follows:

	β	γ
Steel.	—	+
Iron.	—	+
Copper.	+	—
Nickel.	+	—
Brass.	?	?
Nickelin.	—	+
German silver.	—	+

It is not possible to investigate the dependence of the modulus of torsion on tension outside of rather narrow limits. The tension can not exceed the limit of elasticity on the one hand; on the other hand it is not practicable to carry on the investigation with negative tensions, as this would necessitate the use of short thick bars, where a uniform distribution of the stress over the end surface is almost unattainable. Between the tension zero and the elastic limit, the present investigation shows that the law

$$m = \alpha + \beta P + \gamma P^2$$

holds true within the limits of error of the experiments, and that the different substances used behave in the following way: Both steel and iron have negative β and positive γ, which, however, is so small that β alone determines their behavior. Their modulus of torsion decreases therefore, as the tension increases. Copper, and brass with a large percentage of copper, have positive β and negative γ; the latter is large enough to determine their behavior for moderate tensions. The modulus of torsion first increases a little, then decreases quickly. The modulus of torsion reaches a maximum when the tension is about 6 kg. per mm^2. Nickel has positive β and very small negative γ. Its modulus of torsion increases as the ten-

sion increases, as Cantone and Michelucci [1] also found. Nickel is the only one of the substances I investigated, which has this property up to the limit of elasticity. For brass both β and γ are very nearly zero, so that the modulus of torsion varies only slightly with the tension. Nickelin has negative β and very small positive γ; its modulus of torsion decreases with increasing tension. For German silver β is negative, γ positive and small; its modulus of torsion also decreases with the tension. For bronze I have not enough observations to draw conclusions about the universal constants.

To sum up the results of this investigation we may say:

1. That a dependence of torsional elasticity on tension, which is to be expected from the theory, actually occurs.

2. That this dependence can be expressed, within the limits of error of the observations, by the law $m = \alpha + \beta P + \gamma P^2$, where m is the modulus of torsion, P the tension, and α, β, γ are constants.

3. That on account of the inhomogeneity of the surface of the wires, no conclusion about the constants for the substance in question can be drawn from observations on a single wire; but the values of β and γ obtained from such observations represent merely the behavior of the individual wire.

4. That in spite of this, observations of a series of wires of the same substance and different diameters make it possible to draw conclusions about the universal constants for the substance. It is found accordingly that the modulus of torsion of steel, iron, brass (when it does not contain too much copper), nickelin and German silver decreases with increasing tension; that that of copper, and brass containing much copper, at first increases and then decreases; that that of nickel increases continuously. This holds between the tension zero and the limit of elasticity.

[1] Rendiconti della reale Accademia dei Lincei (6), pp. 191–198, 1897.

A DETERMINATION OF THE VISCOSITY OF WATER.

BY E. R. DREW.

THE apparatus used in this investigation is an attempt to realize as nearly as possible the ideal condition from which is obtained the conception of coefficient of viscosity, namely, that of a liquid contained between two parallel planes of infinite extent, one of which is moving in a direction parallel to the other, with constant velocity. It consists essentially of two vertical coaxial cylinders, the outer capable of rotating at any desired speed, while the inner, which is closed at the ends, is suspended by means of a torsion wire between agate bearings in the axis. The liquid used is contained in the space between the two cylinders.

The idea of coaxial cylinders was used for the same purpose by Professor Perry ("Liquid Friction," Phil. Mag., 35–441, 1893), who has given the complete mathematical theory, provided the effect due to the lower end of the suspended cylinder may be neglected or allowed for. The apparatus here used was constructed by Mr. E. L. Johonnott, under the direction of Professor Michelson; and as they were at the time unacquainted with the work of Perry, the idea was carried out in a quite different manner. Mr. Johonnott had completed the apparatus, and obtained some preliminary observations, when his attention was called to other lines of work, and the continuation of the investigation was intrusted to the writer by Professor Michelson.

The main object in view was to obtain a result which could be compared with those which have been obtained by the very different method, due to Poiseuille, of flow through capillary tubes.

Fig. 1 is a section through the axis of the instrument. The inner cylinder, A, is held in position by two agate bearings b, b, by means of which it can be centered with considerable accuracy. The torsion wire is attached to the cylinder by means of the rigid brass

frame *F*, which passes around the heavy cross-bar carrying the upper bearing. When in use, the bearings and suspending wire are so adjusted as to give an amount of side play barely perceptible to the touch, in each bearing. The effects of friction in the bearings are then inappreciable. Deflections are read with telescope and scale, the latter in the form of an arc of a circle about the suspending wire as center.

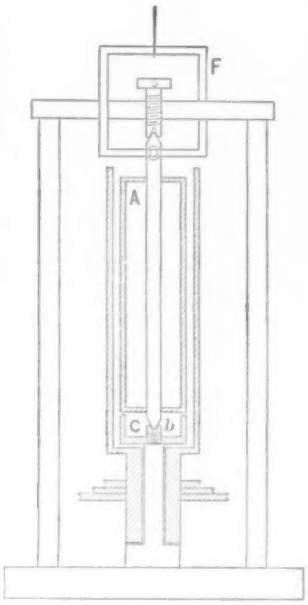


Fig. 1.

The inner cylinder was accurately turned; but as the inner surface of the outer cylinder could not readily be turned, with the facilities at hand, selected pieces of brass tubing which were very nearly cylindrical were used, and an attempt made to experimentally determine the error introduced. The outer cylinder is surrounded by a water-jacket, not shown in the figure, which allows the temperature of the liquid used to be readily determined, and aids in keeping it steady.

An ingenious and simple chronograph attachment, also not shown, was devised by Mr. Johonnott, to record the number of turns per second made by the outer cylinder during the time of an observation. Power was at first obtained from an engine in the mechanician's shop, but the speed was not sufficiently steady, and a 1-h.-p. electric motor was substituted, which, with a heavy balance-wheel on the countershaft, gave good satisfaction.

C is a brass cup screwed on the central shaft, with its upper edge almost in contact with the lower end of the suspended cylinder. It was found in practice that the water which filled the space between the cylinders seldom entered the cup. It was expected that this device would eliminate the effect, previously referred to, due to the lower end of the cylinder. When the cylinder had attained its equilibrium position, due to a steady rotation of the outer cylinder, the cup would form practically a continuation of the cylinder, of con-

siderable length as compared with the thickness of the layer of liquid. Then the effect at a point near the lower end of the cylinder should be nearly or quite the same as at a point on an infinitely long cylinder, and Perry's analysis would apply directly.

A large number of observations showed that the deflection did not increase exactly with the speed, as it should, but somewhat faster. An example will show the magnitude of the effect. The speed is given under n, number of turns per second, and D is the deflection in cm. on the scale.

n	D	D/n
3.63	1.39	.383
5.20	2.01	.387
8.42	3.36	.399
10.61	4.37	.412
13.55	5.72	.422

To see whether this was in any way due to the free surface of the liquid, which was always set just at the upper edge of the suspended cylinder, a cup similar to C was suspended from the cross-bar, to form a continuation of the cylinder, and the free surface of the water raised some distance. The results then obtained showed no appreciable improvement. Subsequent experiments conducted along various lines seemed to indicate that the trouble was at the lower end of the cylinder, but no decisive result could be reached.

A little consideration showed that in order to apply the simple theory, as is done in Perry's discussion, the essential condition which must be fulfilled is that, when steady motion has been attained, all the points in the liquid which have a common speed must lie in a surface which, within the region considered, is everywhere equidistant from the bounding surfaces. Thus the surface of mean speed, in the case of a liquid contained between two planes, is a plane half way between them. In the present case, it is evident that the surface of mean speed will enter the angle P, Fig. 2, at an inclination of about 45° to the vertical, as shown by the dotted line, and will probably be displaced from its proper position for some distance above the lower end of the inner cylinder. If this displacement varies with the speed, the results obtained are accounted for.

The difficulty was overcome by means of the arrangement shown in Fig. 3. The two rings, shown in section at r and s, are attached,

one to the outer cylinder, the other to the cup. The slant face of *s* is then fixed, while that of *r* moves with the outer cylinder, so that the vertical line midway between them represents the surface of mean speed. This is not exactly true, since different portions of the slant face of *r* move with different speeds ; but with the dimensions used,

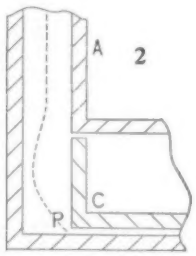


Fig. 2.

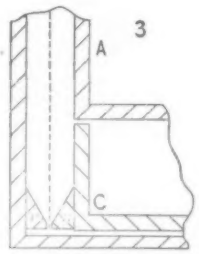


Fig. 3.

the error from this cause is small. With this arrangement the results at different speeds show no consistent difference.

The torsion constant of the suspending wire, defined as the force in dynes which must be applied at the surface of the cylinder, in a direction perpendicular to the axis, to produce a deflection of 1 cm. as read on the scale, was determined in two ways. The first method was that of the torsion pendulum, which took the form of a horizontal brass bar attached at its center to the lower end of the frame *F*. Near its ends, and equidistant from the center, two equal brass cylinders, with their axes vertical, were attached in such a way that they could be easily removed and replaced in the same position.

Following are the data for this determination :

Period of vibration, cylinders in position.	4.131 sec.
" " " " removed.	2.811 "
Moment of inertia of cylinders in position.	13780 gram-cm².
Radius of suspended cylinder *A*.	2.300 cm.
Scale distance.	76.52 cm.
Torsion constant, as previously defined.	161.3 dynes.

The plan of the second method is shown in Fig. 4; *acd* is a very fine silk fiber, one end of which is wound around the upper end of the suspended cylinder, the outer cylinder and the bearings having been removed. The other end is wound around a pin at *a*, and a small mass *m* is hung from *c* by a similar fiber. A third fiber is hung from *a* to serve as a plumb-line. The pin at *a* is attached to

an arm pivoted near *c*, so that different inclinations of *ac* may be used. The portion *cd* of the fiber is brought to the horizontal each time by winding on the pin at *a*. The distances *ab* and *bc* are readon millimeter scales, and the horizontal force calculated. This force pulled the suspending wire out of the vertical, and slightly increased the scale-distance, from 76.52 cm. to 76.70 cm. in the extreme case, and this was allowed for in calculating results.

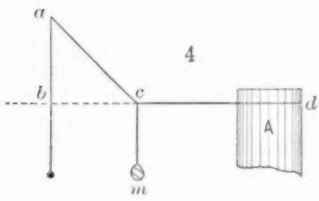


Fig. 4.

Torsion constant, mean of ten determinations, 161.4 dynes.

Greatest difference between two determinations, 0.6 dyne.

Two outer cylinders, of different diameters, were used. As they were not truly cylindrical, 16 equally-spaced diameters were measured at each end.

Dimensions of cylinders.	Outer Cylinder No. 1.	Outer Cylinder No. 2.	Inner Cylinder.
Mean radius, R_1.	2.538 cm.	2.851 cm.	R_2, 2.300 cm.
Greatest difference between two radii.	0.0023 cm.	0.015 cm.	
Length, L.			25.30 cm.

The final determinations were made at temperatures which seldom differed by more than 0°.2 from 20°, and the results were reduced to 20° by the formula computed by Helmholtz from Poiseuille's observations.[1]

$$\mu = \frac{0.0178}{1 + .0337t + .0022t^2}.$$

Observations were made at speeds of about 4, 6 and 8 turns per second. The result computed from each observation is the ratio D/n, the deflection which would be produced at a speed of one turn per second. These results are here summarized:

Outer Cylinder, No. 1.	
Mean of 33 observations.	$D/n = 1.615$
Greatest difference.	0.012
Average deviation from mean.	0.0031
Outer Cylinder, No. 2.	
Mean of 26 observations.	$D/n = 0.821$
Greatest difference.	0.009
Average deviation from mean.	0.0018
Scale distance.	76.18 cm.
Torsion constant of wire reduced to this distance.	160.6 dynes.

[1] See O. E. Meyer, Wied. Ann., 1877, p. 391.

The formula reached by Perry is

$$M = 4\pi\mu\omega R_1^2 R_2^2 / (R_1^2 - R_2^2)$$

where ω is the angular velocity of the outer cylinder, and M is the moment per unit length of the suspended cylinder, exerted upon its surface by a liquid for which μ is the coefficient of viscosity. Adapting this to the present case, it becomes

$$\mu = FDR_1(R_1^2 - R_2^2)/8\pi^2 L R_1^2 R_2^2$$

where D is the deflection at a speed of one turn per second, for which ω is 2π, and F is the torsion constant of the wire. The results are:

Outer cylinder, No. 1, $\mu = 0.01020$ dyne.
Outer cylinder, No. 2, $\mu = 0.01010$ dyne.

The lower end of the outer cylinder slipped somewhat loosely into a sleeve which kept it centered at the bottom, while allowing it to be tipped about slightly, and centered at the top by adjustment. In each case a set of observations was made with the top out of center by a measured amount, in order to obtain an approximate value for the error arising from the fact that the outer cylinder was not truly cylindrical. When the two cylinders are set coaxially, the greatest difference between two radii gives δt, the greatest difference in thickness of the liquid layer. At a given point on the surface of the inner cylinder this change in thickness would occur twice in each revolution. When the outer cylinder is tipped so that its upper end is off center, the greatest change in thickness, $\delta_1 t$, occurs but once in each revolution. Assuming that for such small variations, the error is proportional to the first power of the effective δt, the error in the first case would be proportional to $2\delta t$, and in the second to $2\delta t + \delta_1 t/2$. In the table of results, D_1 is the deflection obtained when the outer cylinder was tipped.

	D	D_1	δt	$\delta_1 t$
Outer cylinder No. 1.	1.615	1.595	0.0023 cm.	0.03 cm.
" " " 2.	0.821	0.813	0.0150 cm.	0.05 cm.

This gives the following corrections to be applied to the results:

	Correction.	Corrected μ.
No. 1.	+ 0.3 per cent.	0.01023
" 2.	+ 1.0 " "	0.01020

The thermometer used had not been calibrated, and Professor Ames, of Johns Hopkins University, very kindly arranged to have it compared with a Tonnelot standard thermometer. The comparison, made by Mr. Chas. Waidner, showed that at 20°, the temperature of the above determinations, the thermometer differed from the Paris nitrogen standard by 0°.03, somewhat less than the probable error of a reading. So that no temperature correction is necessary.

To compare with these results, the value of the coefficient from the observations of Poiseuille, computed by the formula previously given, is $\mu = 0.0101$.

RYERSON PHYSICAL LABORATORY, UNIVERSITY OF CHICAGO, March, 1898.

THE APPLICATION OF THE MANOMETRIC FLAME TO THE TELEPHONE.

By Louis W. Austin.

A CONVENIENT method of showing visually the complex motions of the diaphragm of a telephone receiver seems of some considerable scientific interest. The methods which have been used for studying these vibrations are for the most part complicated and difficult to carry out, and do not any of them, so far as I know, permit the accurate study of vibrations of any degree of complexity.

The great sensibility of the manometric flame to small changes of gas pressure is well known and since the possibility of preserving its records photographically has been shown by Professor Merritt [1] the field of its usefulness has been greatly increased. This great sensibility suggests the possibility of its use in making visible the sound waves produced in the telephone. In order to test the feasibility of this idea a telephone receiver was connected to an ordinary manometric capsule by means of a rubber tube passing through a cork cemented into the opening in the telephone cap. The telephone was then connected in a shunt from the alternating circuit and about .01 ampère sent through it. This made the flame dance excitedly, but when examined in the revolving mirror the image showed very little resemblance to the known form of current wave. The real cause of these inaccuracies was not at first apparent. Several different jets and capsules were used and changes were made in the distance between the telephone diaphragm and the cap, as it was thought that perhaps the diaphragm might come in contact with the cork in the opening, but it was not until the connecting tube was done away with that any improvement was discovered. In the new arrangement a jet was inserted directly through the rubber cork in the telephone cap and another hole drilled in the cap

[1] Ernest Merritt, Phys. Rev., Vol. I., pp. 166, 1893.

through which the gas was introduced, thus making a manometric capsule of the telephone itself. These changes seemed decidedly beneficial, showing that at least a large part of the difficulty lay in the friction and elasticity of the connecting tube. As it was thought possible that the remaining inaccuracies might be due to pulsations set up in the rubber tube introducing the gas, the gas opening in the cap was constricted to about the size of the opening in the jet. After this change had been made, the lower edge of the luminous part of the flame as seen in the revolving mirror assumed nearly the sine form, and the sensibility of the apparatus was increased in a marked degree. Fig. 1 shows the general arrangement of the telephone in its final form, *A* being the jet passing through the cork *C*, and *B* the inlet for the gas.

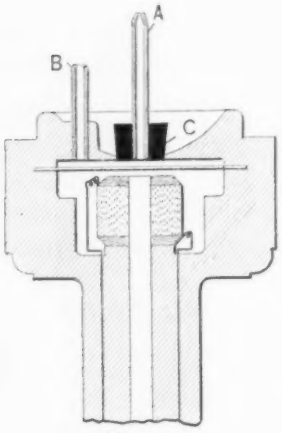


Fig. 1.

The apparatus has been tried with a number of different alternators and the curves from all of these seem to show their known peculiarities of form, though it must be confessed that a certain distortion seems to be present. The reason for this has not been with certainty determined, but it seems probable that it is due at least in part to the flame itself and its tendency to flare.

In these earlier observations drawings of the wave forms were made, but the fleeting character of the images made their reproduction in this way difficult and somewhat uncertain, and it was therefore decided to try photography. As the ordinary illuminating gas does not give a flame which is actinic enough for the purpose unless mixed with oxygen as suggested by Professor Merritt, acetylene was tried. This was found to be highly satisfactory photographically, but the distortion seemed to be increased so that the photographed curves are probably much less accurate than those observed in the revolving mirror when the illuminating gas was used. The photographs were taken with an ordinary camera which was swung around in front of the flame on a kind of whirling table, the camera being placed at such a distance as to make the image not far from the same size as the flame.

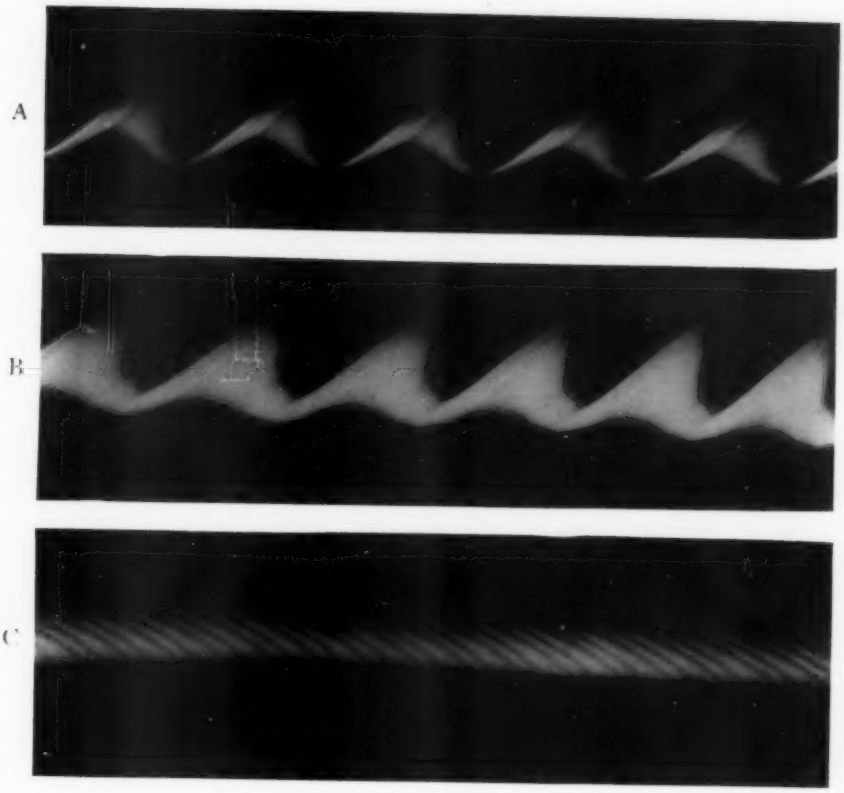


FIG. 2 (AUSTIN)

In Fig. 2, *a*, is shown a photograph of the current curve of the Madison day incandescent lighting system, with a flame about 15 mm. high. The flaring tendencies are here much worse than in the case of the illuminating gas, the left side of each wave being especially distorted from this cause. In *b* the same current curve is shown photographed with a flame 8 mm. high and with a smaller current through the telephone. This seems to be less distorted, but at the same time loses some of the distinctness of its characteristics.

It was found here as well as in the case of the illuminating gas that when a certain amplitude of vibration of the telephone diaphragm was exceeded the flame curve became discontinuous and gave out a loud humming sound reproducing the note of the transformer, this critical amplitude being apparently roughly proportional to the height of the flame.

The wave forms produced by speaking into a carbon transmitter were also observed. It was found that any tone which sounded strong and loud when heard in a receiver produced a well-marked motion in the flame, but as was to be expected the latter did not respond to the weaker sounds. In Fig. 2, *c*, is shown a photograph of the flame when the vowel a, pronounced as in far, was spoken into the transmitter. The flame in this case was about 1 cm. high. It seems probable that the small undulations of the bottom of the flame, which are quite clear with the illuminating gas but unfortunately indistinct in the photograph, represent the real wave form of the sound.

While it had been already shown that the lower line of the luminous part of the flame followed quite accurately the motions of the diaphragm, it was thought worth while to investigate what degree of proportionality existed between the rise in the top of the flame and the potential difference across the terminals of the telephone. To try this the following experiment was carried out: An alternating current of about 0.8 ampère was sent through a bare wire so as to produce a fall of potential of about 0.07 volt per cm. along its length. From this various voltages were applied to the telephone, it being assumed that the motion of the telephone diaphragm could be considered proportional to the difference of potential at its terminals. A millimeter scale was mounted close by the flame and

both were observed through a large lens magnifying about three diameters. This enabled readings to be taken to 0.1 mm. with considerable certainty. Observations were made with flames of different heights both in the case of illuminating gas and acetylene. Jets of four different-sized openings were used, 0.28 mm., 0.6 mm., 1.1 mm., 1.8 mm. With illuminating gas it was found that the rise in the top of the flame was, as nearly as could be observed, proportional to the voltage, up to about the point where the buzzing of the flame became quite noticeable. It was also found to be nearly independent of the original height of the flame up to 50 mm. in the case of the two smaller jets. The smallest jet, 0.28 mm., was found to be less than half as sensitive as the others, while the two largest seemed less regular in their action, therefore 0.6 mm. was chosen as the best size for use. With this the rise in the flame was about 6 mm. per volt. The low acetylene flame behaved much the same as the illuminating gas, but the high flame seemed to increase in height more rapidly than the voltage.

During the observation it was noticed that the lower part of the luminous portion of the illuminating gas flame when vibrating appeared to draw itself down in the form of an inverted cup which was very sensitive to slight changes in voltage, moving up and down constantly, apparently at each revolution of the engine.

The above observations suggest several possible lines of usefulness for the manometric telephone.

1. It may be used to show in the lecture room the motions of the telephone diaphragm, and illustrate the undulations of an alternating current.

2. It may, perhaps, be developed so as to offer a practical and convenient method of tracing alternating current curves.

3. In connection with a second manometric flame actuated by a tuning fork of known period it becomes a frequency teller.

4. It may be used in studying the distortions in sounds, produced by telephone transmission.

5. It offers a convenient method of comparing small alternating electromotive forces.

THE UNIVERSITY OF WISCONSIN, Dec., 1900.

MOUNTING FOR TUBE CONTAINING LIQUEFIED CO_2.

By Edwin H. Hall.

IN Fig. 1, which shows a vertical section of the apparatus, L is a massive leaden base supporting a vertical pillar which carries a strong brass tube BB. Each end of this tube is closed by a screw cap, the flat inner surface of which is covered with cork. Perforated plugs of cork, the top one of which is marked c, hold the CO_2 tube, tt, secure in the upright position. T is a thermometer the bulb of which is near the meniscus in tt. V_1, supported at any desired height by the arm A, is a

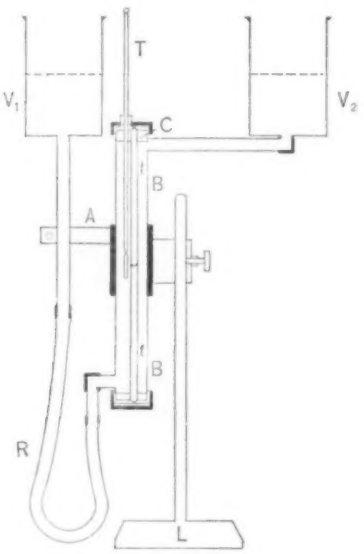


Fig. 1.

Fig. 2.

vessel containing water at, let us say, 27° C. V_1 has communication through the rubber tube R and through BB, with a similar vessel V_2, containing water near 37° C. The temperature of the water in BB is of course intermediate.

In Fig. 2, which shows a horizontal section through the middle of VB, the arrows indicate the line of sight, which passes through two thick plates of glass. Each glass plate is held in place by a strong flanged frame of brass, fastened by screws not shown.

As the meniscus moves up and down a centimeter or more, it is well to make the glass windows three or four centimeters long.

To make the meniscus vanish it is only necessary to lower V_1 a few centimeters, whereupon the warmer water, flowing down from V_2 into BV, so heats the tube tt that the liquid part of its contents expands considerably and then becomes indistinguishable from the vapor. To restore the meniscus, raise V_1 a few centimeters above the level shown in the figure.

NEW BOOKS.

(1) *Polyphase Electric Currents and Alternate-Current Motors.* (Second, enlarged edition.) By SILVANUS P. THOMPSON. New York, Spon & Chamberlain, 1900.

(2) *Theory and Calculation of Alternating Current Phenomena.* (Third edition, revised and enlarged.) By CHARLES PROTEUS STEINMETZ, with the assistance of ERNST J. BERG. New York, *Electrical World and Engineer*, 1900.

(3) *Standard Polyphase Apparatus and Systems.* (Second edition.) By MAURICE A. OUDIN. Pp. 242. New York, D. Van Nostrand Co., 1900.

These are new editions of three useful books relating to different phases of a common subject. Professor Thompson's book has been revised and exactly doubled in size, the 244 pages of the first edition being expanded to 488 in the second. Some changes are made in the original matter and material has been added on induction motors. Chapters VII. and VIII., on the Graphic Theory of Polyphase Motors, have been developed by Mr. Miles Walker, who, together with Mr. A. C. Eborall, has assisted in the revision of the work. Eight folding plates of scale drawings are included and colored illustrations have been made use of in the text to make clear the diagrams of polyphase windings.

The third edition of Mr. Steinmetz's work has shown less expansion—namely, from 424 pages in the first edition to 512 in the third—but its value has been very much enhanced by a careful revision by which errors have been reduced; certain chapters have been rewritten and the treatment made more systematic as to notation. The thorough overhauling has greatly improved the book. To the thirty chapters in the first edition only two have been added, namely, Chapter XII., "Power and Double Frequency Quantities in General," and XIV., "Symbolic Representation of Alternating Wave," these chapters occupying 28 pages. The chapters on topographical method, on distributed capacity and inductance, and on frequency converters and induction machines have been revised and enlarged; only minor changes being made in the remainder of the book.

The work by Ouden apparently differs from the first edition chiefly in its revision.

These three works encroach but little upon each other. Ouden treats of the material—the apparatus—which forms the subject matter for any theoretical discussion. The writer shows a wide familiarity with his subject, both in theory and in practice, but limits himself exclusively to a discussion of apparatus and systems. Naturally much of this information can be gained from trade publications, but not set forth in so useful a manner. The book contains some careless inaccuracies of expression, not enough however to seriously detract from its worth; it is valuable for one who is studying the theory of alternating currents and desires information concerning the concrete machines to which this theory applies.

Steinmetz's work is a classic and contains a wealth of original material which is rarely approached in any book of its class. For the most part it is abstract, and much study from the reader is required for its mastery. The engineer or student who grasps its methods has acquired indeed a valuable tool.

Thompson's book is written in the same easy style which characterizes his other works; the apparent simplicity often compensates for lack of rigor. It falls between the other two works in style, containing descriptions of apparatus and theory combined. It familiarizes the reader with a subject not treated with equal fullness elsewhere.

F. BEDELL.

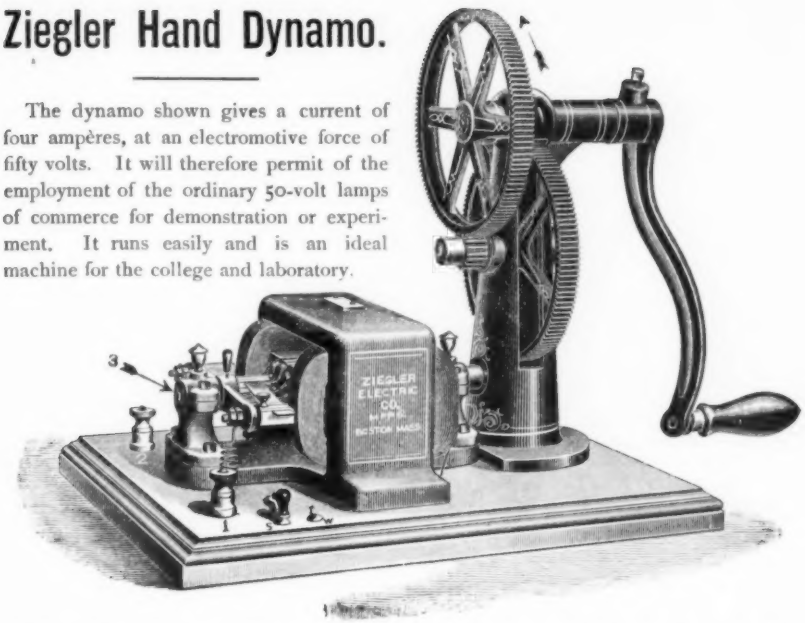
Ames's Scientific Memoirs, No. 10. The Wave Theory of Light. Memoirs by HUYGENS, YOUNG & FRESNEL, edited by HENRY CREW. New York, The American Book Co., 1900. Pp. xv + 164.

This little volume is the first of the well-known series of scientific memoirs begun under the directorship of Professor Ames, and hitherto published by Harper Bros., to be issued by the American Book Co. In it the editor, Professor Crew, has done something more than merely to reproduce and bring together in a convenient form the first three chapters of Huygens's Treatise on Light, Young's three papers (entitled, "Theory of Light and Color, The Production of Colors not Hitherto Described and Experiments and Calculations Relative to Physical Optics") and Fresnel's celebrated memoir on the diffraction of light. He has in a short but exceedingly interesting and suggestive preface pointed out the relation of these works to the earlier investigations of Robert Hooke and to the work of Newton on the one hand and the bearing of the later researches of Fizeau, Foucault, Michelson and Morley, Stokes, Maxwell, Hertz and Zeeman on the other. The three brief biographical sketches which follow the papers of Huygens, Young and Fresnel respectively are good reading and the bibliography with which the volume closes, while by no means complete, offers a useful list of some of the more important transactions and memoirs dealing with undulatory theory of light.

E. L. N.

Ziegler Hand Dynamo.

The dynamo shown gives a current of four ampères, at an electromotive force of fifty volts. It will therefore permit of the employment of the ordinary 50-volt lamps of commerce for demonstration or experiment. It runs easily and is an ideal machine for the college and laboratory.



ZIEGLER ELECTRIC CO., 141 Franklin St., BOSTON, MASS.

Send for a copy of Physical Catalogue No. 10.

Recent Text-Books on Mechanics

By FREDERICK SLATE

The Principles of Mechanics

An Elementary Exposition for students of Physics of Kinematical and Dynamical Mechanics.

By FREDERICK SLATE, Professor of Physics, University of California. Part I., pp. x + 299.
Cloth, 12mo, **$1.90** *net.*

The needs of college students at an elementary stage are considered; but also the value of Mechanics as a system of organized thought, of distinct culture value.

By Sir ROBERT S. BALL

A Treatise on the Theory of Screws

An exhaustive thoroughly illustrated Treatise.

By Sir ROBERT STAWELL BALL, LL.D., F.R.S., Lowndean Professor of Astronomy and Geometry in the University of Cambridge.
Cloth, 8vo, **$5.50** *net.*

THE MACMILLAN COMPANY

66 FIFTH AVENUE NEW YORK

A History of Physics

In Its Elementary Branches, including the Evolution of Physical Laboratories

By FLORIAN CAJORI, Ph.D.

Professor of Physics in Colorado College.

Cloth Cr. 8vo, $1.60 *net.*

This brief popular History gives in broad outline the development of the science of physics from antiquity to the present time. It contains also a more complete statement than is found elsewhere of the evolution of physical laboratories in Europe and America. The book, while of interest to the general reader, is primarily intended for students and teachers of physics. The conviction is growing that, by a judicious introduction of historical matter, a science can be made more attractive. Moreover, the general view of the development of the human intellect which the history of a science affords is in itself stimulating and liberalizing.

"Prof. Cajori has followed his 'History of Mathematics' with a 'History of Physics' that is *even more interesting and valuable.* . . . Every teacher and student of physics should read it and add it to his library. It shows careful study, enthusiasm and a comprehensive grasp of the subject."—*Journal of Education.*

"A most valuable contribution to the subject."—*Scientific American.*

"Probably this will prove the most successful of all Prof. Cajori's histories."
—*The Nation.*

BY THE SAME AUTHOR

A History of Mathematics

Cloth 8vo, $3.50 *net.*

"What we have a right to expect in such a handbook is an agreeable narrative of the most material events in the history of mathematics, and this Professor Cajori incontestably supplies. The book was much wanted."—*The Nation.*

"A scholarship both wide and deep is manifest in this *History of Mathematics* which the author has infused with his own ardor in this department of science."
—*Journal of Education.*

"To the student with a love for mathematical science this book will be as entertaining as a romance."
—*The Transcript*, Boston.

A History of Elementary Mathematics

Cloth 12mo, $1.50 *net.*

"A most instructive, and at the same time a very readable piece of work, full of curious facts."—*The Bookman.*

"By no means an abridged edition of *The History of Mathematics.* It is an entirely new book, giving a somewhat detailed account of the rise and progress of Arithmetic, Algebra and Geometry. The book should be read by all teachers of these subjects, and by mathematical students generally."
—*American Mathematical Monthly.*

"The product of wide and scholarly research . . . For its historical facts and its suggestiveness, this work should be read carefully by all students and teachers of mathematics."—*The Dial*, Chicago.

THE MACMILLAN COMPANY, Publishers, New York

New Books on Astronomy, Optics, etc., etc.

Among THE MACMILLAN COMPANY'S Publications

By Sir ROBERT BALL, LL.D.

The Elements of Astronomy

By the author of "Starland," "Atlas of Astronomy," "The Story of the Sun," "The Theory of Screws," etc., etc.

By Sir ROBERT BALL, LL.D., Lowndean Professor of Astronomy and Geometry in the University of Cambridge, formerly Royal Astronomer of Ireland. Cloth, illustrated, **80 cents** *net.*

By JOHN COUCH ADAMS

Lectures on the Lunar Theory

The Lectures of Prof. Ball's predecessor, edited by R. A. SAMPSON, University of Durham.

By JOHN COUCH ADAMS, M.A., F.R.S., late Lowndean Professor of Astronomy and Geometry in the University of Cambridge. Cloth, 8vo, **$1.25** *net.*

By W. HASTIE, D.D.

Kant's Cosmogony

Kant's Essay on the Retardation of the Rotation of the Earth and his Natural History and Theory of the Heavens.

Edited and Translated by W. HASTIE, D.D., Professor of Divinity, University of Glasgow. With an Introduction and Appendices by and a portrait of THOMAS WRIGHT, University of Durham. Cloth, cr. 8vo, **$1.90** *net.*

By R. A. HERMAN

A Treatise on Geometrical Optics

An extended treatise with index, diagrams, etc.

By R. A. HERMAN, M.A., Fellow and Lecturer of Trinity College, Cambridge. Cloth, 8vo, **$3.00** *net.*

By Sir NORMAN LOCKYER, K.C.B.

Recent and Coming Eclipses

Accounts of Observations in India, 1898, with Conditions of Eclipses of 1900, 1901 and 1905.

By Sir NORMAN LOCKYER, K.C.B., F.R.S., author of "The Sun's Place in Nature," etc. *Second Edition.* Illustrated. Cloth, 8vo, **$2.00** *net.*

Send for our latest classified catalogue. Address,

THE MACMILLAN COMPANY, 66 Fifth Ave., New York

Standard Text-Books on General Physics

The Elements of Physics

FOR USE IN HIGH SCHOOLS.

By HENRY CREW, Ph.D., Northwestern University.

Second Edition, Revised, **Cloth. $1.10,** *net.*

Elements of Theoretical Physics

By DR. C. CHRISTIANSEN

Professor of Physics, University of Copenhagen

TRANSLATED INTO ENGLISH BY

W.F. MAGIE, Ph.D., Professor of Physics, Princeton University **Cloth. 8vo. $3.25, net.**

Problems and Questions in Physics

CHARLES P. MATTHEWS, M.E., AND JOHN SHEARER, B.S.

Purdue University *Cornell University*

8vo. Cloth. pp. 247 + 4. Price $1.60, net

An Intermediate Course of Practical Physics

By A. SCHUSTER, Ph.D., F.R.S., AND C. H. LEES, D.Sc.

Owens College, Manchester *Owens College, Manchester*

12mo. Cloth. pp. xv + 248. Price $1.10 net.

Lessons on Elementary Practical Physics

By BALFOUR STEWART, A.M., LL.D., F.R.S., AND W. W. HALDANE GEE

Vol. I. **General Physical Processes.** 12mo. $1.50, *net*

Vol. II. **Electricity and Magnetism.** $2.25, *net*

Vol. III. Part I. **Practical Acoustics.** $1.10, *net*

Part II. **Heat and Light.** *In Press*

Laboratory Manual of Physics and Applied Electricity

Arranged and Edited by EDWARD L. NICHOLS

Professor of Physics in Cornell University

IN TWO VOLUMES **Vol. I. Cloth. Price $3.00, net**

JUNIOR COURSE IN GENERAL PHYSICS. By ERNEST MERRITT and FREDERICK J. ROGERS. **Vol. II. Cloth. pp. 444. Price $3.25, net**

SENIOR COURSES AND OUTLINES OF ADVANCED WORK By GEORGE S. MOLER, FREDERICK BEDELL, HOMER S. HOTCHKISS, CHAS. P. MATTHEWS, and THE EDITOR.

A large proportion of the students for whom primarily this Manual is intended are preparing to become engineers, and special attention has been devoted to the needs of that class of readers.

THE MACMILLAN COMPANY New York: 66 Fifth Avenue

BOSTON CHICAGO SAN FRANCISCO

IMPORTANT PUBLICATIONS

HISTORY OF MODERN PHILOSOPHY IN FRANCE. With twenty-three Portraits of French Philosophers. By Professor L. LÉVY-BRUHL, Maître de Conférences in the Sorbonne, Paris. Handsomely bound. Pages, 500. Price, **$3.00** (12s.).

THE HISTORY OF THE DEVIL AND THE IDEA OF EVIL. FROM THE EARLIEST TIME TO THE PRESENT DAY. By DR. PAUL CARUS. Printed in two colors from large type on fine paper Bound in cloth, illuminated with cover stamp from Doré. 500 8vo pages, with 311 illustrations in black and tint. Price, **$6.00** (30s).

SCIENCE AND FAITH, OR MAN AS AN ANIMAL AND MAN AS A MEMBER OF SOCIETY. WITH A DISCUSSION OF ANIMAL SOCIETIES. By DR. PAUL TOPINARD, Late General Secretary of the Anthropological Society of Paris, and Sometime Professor in the School of Anthropology. Pp., 361. Price, cloth, gilt top, **$1.50** (6s. 6d.).

"A most interesting volume."—*Glasgow Herald.*
"Stimulating and suggestive."—*The Scotsman.*

THE EVOLUTION OF GENERAL IDEAS. By TH. RIBOT, Professor in the Collège de France. Authorized translation from the French by FRANCES A. WELBY. Pp., xi, 231. Price, cloth, **$1.25** (6s. 6d.).

"All that he writes is lucid and suggestive, and the course of lectures here translated is a characteristic contribution to psychology."—*Nature.*

ELEMENTARY ILLUSTRATIONS OF THE DIFFERENTIAL AND INTEGRAL CALCULUS. By AUGUSTUS DE MORGAN. New reprint edition. With sub-headings and bibliography of English and foreign works on the Calculus. Red Cloth. Price, **$1.00** (4s. 6d.).

"The present work may be safely recommended to those students who are anxious to obtain a knowledge of the Calculus which shall be real and abiding."
—*The Speaker*, London.

"It aims not at helping students to cram for examinations, but to give a scientific explanation of the rationale of these branches of mathematics. Like all that De Morgan wrote, it is accurate, clear, and philosophic."—*Literary World.*

"It would be difficult to overestimate the value of De Morgan's works and of the importance of rigid accuracy upon which he everywhere insists."
—*The Speaker*, London.

A BRIEF HISTORY OF MATHEMATICS. An Authorized Translation of Dr. Karl Fink's *Geschichte der Elementar-Mathematik.* By WOOSTER WOODRUFF BEMAN and DAVID EUGENE SMITH. With biographical notes and full index. Pages, 333+xii. Cloth, **$1.50** (5s. 6d.).

"Dr. Fink's work is the most systematic attempt yet made to present a compendious history of mathematics."—*The Outlook.*

THE OPEN COURT PUBLISHING COMPANY

324 Dearborn Street, Chicago, Ill.

LONDON: Kegan Paul, Trench, Trübner & Co., Ltd.

The Astrophysical Journal

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE

Director of the Yerkes Observatory

WITH THE COÖPERATION OF

THE LEADING ASTROPHYSICISTS OF THE UNITED STATES AND EUROPE

Contents of Recent Numbers

JULY 1900.

On the Spectrum of Radium . . C RUNGE

The Velocity of Meteors as Deduced from Photographs at the Yale Observatory W. L ELKIN

Some New Fluorescence and Afterglow Phenomena in Vacuum Tubes Containing Nitrogen . . . . . PERCIVAL LEWIS

The Effect of Some Impurities on the Spectra of Some Gases. II. . . PERCIVAL LEWIS

Prominences Observed at Kalocsa on May 28, 1900 . . . . . J. FÉNYI

A Suggested Explanation of the Solar Corona . . . . . J. SCHEINER

Remarks on the Construction and Adjustment of Spectrographs. II. J. HARTMANN

Note on the Spectrum of Silicon FRANZ EXNER and EDUARD HASCHEK

Pressure in the Spark EDUARD HASCHEK and HEINRICH MACHE

James Edward Keeler . . . GEORGE E. HALE

Minor Contributions and Notes: A New Star in *Aquila*, E. C. PICKERING; The Yerkes Observatory of the University of Chicago, *Bulletin* No 13, GEORGE E HALE; Positions of *Eros* (433) in 1893, 1894, and 1896 EDWARD C. PICKERING

Eclipse Reports: Preliminary Results of the United States Naval Observatory Eclipse Expeditions, S. J. BROWN; A Preliminary Statement of the Results of the Smithsonian Observatory Eclipse Expedition, C. G. ABBOT; Eclipse Observations by the Princeton Party at Wadesboro, N. C., May 28, 1900, C. A. YOUNG; The Yerkes Observatory of the University of Chicago, *Bulletin* No. 14, Observations of the Total Solar Eclipse of May 28, 1900, at Wadesboro, N. C., GEORGE E. HALE; Observations of the Total Eclipse at Centerville, Norfolk County, Va, WINSLOW UPTON; Observations at Juliette, Ga., C W. CROCKETT; Observations of the Total Solar Eclipse of May 28, 1900, by the Harvard Observatory Expedition, WILLIAM H. PICKERING; Observations of the Total Eclipse by the Vassar College Party, MARY W. WHITNEY; Observations by the Massachusetts Institute of Technology Party, ALFRED E. BURTON; Observations at Wadesboro, N. C., J B. COIT; Observations at Southern Pines, N C, F. E. SEAGRAVE; The Weather Bureau Observations of the Total Eclipse of May 28, 1900; Observations at Norfolk, Va., FRANK W. VERY; Observation by the Georgetown College Party, J. G. HAGEN; Persistence of the Corona After Totality, JOHN FRED MOHLER.

SEPTEMBER 1900.

The Spectra of Mercury WILLIAM B. HUFF

An Investigation on the Zeeman Effect HERBERT M. REESE

A New Theory of the *Milky Way* C. EASTON

Minor Contributions and Notes: Variable Stars in Clusters. Rate of Increase of Light, EDWARD C. PICKERING; The Yerkes Observatory of the University of Chicago, *Bulletin* No. 15, GEORGE E. HALE; Comparison of a Prism and a Grating Spectroscope, N. ERNEST DORSEY.

OCTOBER 1900.

On the Arc-Spectra of Some Metals as Influenced by an Atmosphere of Hydrogen HENRY CREW

Some Abnormal Stars in the Cluster *M 13 Herculis* . . . . . E. E. BARNARD

Discovery and Period of a Small Variable Star in the Cluster *M 13 Herculis* E. E. BARNARD

Solar Phenomena, Considered in Connection with Anomalous Dispersion of Light W. H. JULIUS

Note on Inquiries as to the Escape of Gases from Atmospheres G. JOHNSTONE STONEY

The Complete Emission Function P. G. NUTTING

Rise of a Large Prominence on June 1, 1900 J. FÉNYI

Field of the Reflecting Telescope S. C. REESE

Reviews: A general catalogue of 1290 double stars discovered from 1871 to 1899 by S. W. Burnham, S. W. BURNHAM (E. E. Barnard); Photometric Revision of the Harvard Photometry during the years 1891–1894, EDWARD C. PICKERING (J. A. P.)

JANUARY 1901.

On the Period of the Solar Spots SIMON NEWCOMB

On an Apparatus for the Laboratory Demonstration of the Doppler-Fizeau Principle . . . . . A. BÉLOPOLSKY

The Radiation of a Black Body C. E. MENDENHALL and F. A. SAUNDERS

Examination of *Pleiades* and *Eros* Plates Taken with the Crossley Reflector of the Lick Observatory

On the Production of a Line Spectrum by Anomalous Dispersion, and its Application to the "Flash Spectrum" R.W.WOOD

The Nature of the Solar Corona R.W.WOOD

A Preliminary Determination of the Motion of the Solar System . . W. W. CAMPBELL

The Motion of ζ *Geminorum* in the Line of Sight . . . . . W. W. CAMPBELL

Some Stars with Large Radial Velocities W. W. CAMPBELL

CHICAGO

The University of Chicago Press

LONDON: WM. WESLEY & SON

Published every month except July and September

Subscription Price
Domestic, $4.00; Foreign, $4.50

PORTRAITS

To satisfy a demand which has arisen, the publishers have arranged to issue a limited number of copies of the portraits of scientific men, that have appeared from time to time as frontispieces in the PHYSICAL REVIEW.

The portraits are printed in photogravure, on plate paper, suitable for framing, and are for sale at the very low price of 25 cents each.

NOW READY.

JOHN TYNDALL
HEINRICH HERTZ
HERMAN VON HELMHOLTZ (1876)
AUGUST KUNDT
ALFRED M. MAYER
WILLIAM A. ROGERS
GUSTAV WIEDEMANN
ROBERT BUNSEN
THOMAS PRESTON

THE PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics

CONDUCTED BY

EDWARD L. NICHOLS, ERNEST MERRITT and FREDERICK BEDELL

LVIII. November-December, 1900

TABLE OF CONTENTS

Effect on the Persistence of Vision of Exposing the Eye to Light of Various Wave-Lengths. FRANK ALLEN.

Gibbs' Thermodynamical Model for a Substance Following Van der Waals' Equation. W. P. BOYNTON.

On the Forms of Curves Presented by the Michelson Interferometer. J. C SHEDD.

New Books: *Ewing:* Strength of Materials.

Index.

LIX. January, 1901

TABLE OF CONTENTS

False Spectra from the Rowland Concave Grating. THEODORE LYMAN.

A Modification of the Usual Method of Determining Transference Numbers and an Investigation of the Influence of the Concentration on their Values in the Case of Tri-Ionic Salts. ARTHUR A. NOYES.

Determination of Poisson's Ratio by Means of an Interference Apparatus. J. R. BENTON.

The Use of the Bicycle Wheel in Illustrating the Principles of the Gyroscope. CHAS. T. KNIPP.

An Automatic Temperature Regulator. CHAS. T. KNIPP.

The Specific Inductive Capacity of Dielectrics in a Magnetic Field. EDWARD C. ROBERTS.

Note on the Effect of Tension on the Permanent Torsion of a Wire. J. R; BENTON.

New Books: *Burbury:* Kinetic Theory of Gases; *Walker:* Physical Chemistry; *Van't Hoff:* Chimie Physique; *Tait:* Scientific Papers; *Ball:* Theory of Screws; *Descartes:* Method of Conducting Reason; *De Morgan:* Calculus; *Ribot:* Evolution of General Ideas; *Bell:* Electric Power Transmission; *Lummis:* Management of Dynamos; *Ostwald's Klassiker, No. 110.*

PUBLISHED FOR CORNELL UNIVERSITY

THE MACMILLAN COMPANY